# Maximizing the Guarded Boundary of an Art Gallery is APX-complete

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**Abstract.** In the Art Gallery problem, given is a polygonal gallery and the goal is to guard the gallery's interior or walls with a number of guards that must be placed strategically in the interior, on walls or on corners of the gallery. Here we consider a more realistic version: exhibits now have *size* and may have different costs. Moreover the meaning of guarding is relaxed: we use a new concept, that of *watching* an expensive art item, i.e. overseeing a *part* of the item. The main result of the paper is that the problem of maximizing the total value of a guarded weighted boundary is APX-complete. This is shown by an appropriate gap-preserving reduction from the MAX-5-OCCURRENCE-3-SAT problem. We also show that this technique can be applied to a number of maximization variations of the art gallery problem. In particular we consider the following problems: given a polygon with or without holes and $k$ available guards, maximize a) the *length of walls* guarded and b) the *total cost* of paintings *watched* or *overseen*. We prove that all the above problems are APX-complete.

## 1 Introduction

In the Art Gallery problem (as posed by Victor Klee during a conference in 1976), we are asked to place a minimum number of guards in an art gallery so that every point in the interior of the gallery can be seen by at least one guard.

Besides its application of guarding exhibits in a gallery, the Art Gallery problem has applications in wireless communication technology (mobile phones, etc): place a minimum number of stations in a polygonal area so that any point of the area can communicate with at least one station (two points can communicate if they are mutually visible).

Many variations of the Art Gallery problem have been studied during the last two decades [2–4]. These variations can be classified with respect to where the guards are allowed to be placed (e.g. on vertices, edges, interior of the polygon) or whether only the boundary or all of the interior of the polygon needs to be guarded, etc. Most known variations of this problem are NP-hard. Related problems that have been studied are MINIMUM VERTEX/EDGE/POINT GUARD for polygons with or without holes (APX-hard and $O(\log n)$-approximable [1, 5, 6]) and MINIMUM FIXED HEIGHT VERTEX/POINT GUARD ON TERRAIN ($\Theta(\log n)$-approximable [5, 6, 8]). In [13] the case of guarding the walls (and not necessarily

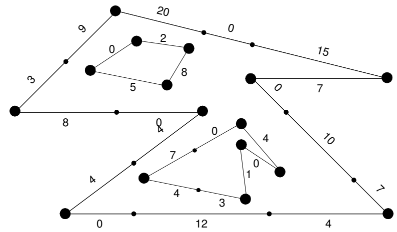


**Fig. 1.** A weighted polygon

every interior point) is studied. In [14] the following problem has been introduced: suppose we have a number of valuable treasures in a polygon $P$; what is the minimum number of mobile (edge) guards required to patrol $P$ in such a way that each treasure is always visible from at least one guard? In [14] they show NP-hardness and give heuristics for this problem. In [15] weights are assigned to the treasures in the gallery. They study the case of placing one guard in the gallery in such a way that the sum of weights of the visible treasures is maximized. Recent (non-)approximability results for art gallery problems can be found in [1–5, 8]. For a nice survey of approximation classes and important results the reader is referred to [11].

Here we consider the Maximum Value Vertex Guard problem: A polygon without holes is given with weighted disjoint line segments on its boundary (see figure 1); an integer $k > 0$ is also given. The goal is to place at most $k$ guards on vertices of the polygon so that the total weight of line segments visible by the guards is maximized. If we think of the weighted line segments as paintings on the walls of an art gallery then we have a realistic abstraction of the problem of guarding a maximum total value of paintings that takes into account the fact that paintings actually occupy parts of the walls, not merely points. Another possible application of this problem is the illumination of a maximum number of paintings in a gallery. Again, a painting must be totally visible from light sources in order to consider it illuminated. There are also important applications in wireless communication networks: An interpretation of weighted line segments are inhabited areas. The polygon models the geographical space. The weight interpretation is the population of an area. Imagine a number of towns lying on the boundary of a polygonal geographical area. The goal is to place at most $k$ stations such that the total number of people that can communicate is maximized. Moreover, it could be the case that the towns are on the shore of a lake, so we can only place stations on the boundary. Similar situations may arise in various other types of landscape.

We show APX-hardness of Maximum Value Vertex Guard and conclude that this problem is APX-complete since there exists a polynomial time constant-ratio approximation algorithm ([12]). Our main contribution is a gap-preserving reduction from Max-5-occurrence-3-Sat to Maximum Value

Vertex Guard specially designed for weighted maximization problems. The construction part of our reduction uses some ideas from the constructions used in [1], [6] (to show NP-hardness, APX-hardness respectively of the Minimum Vertex Guard problem). Central in our technique is a careful assignment of appropriate weights on the line segments of the constructed polygon.

Next we study a number of variations: a) the case of edge guards (guards occupying whole edges), b) the case in which our goal is to *watch* (see a part of) line segments instead of overseeing them, c) the case of maximizing the total length of the visible boundary and e) the case of polygons with holes. We prove APX-completeness for all these variations and for several of their combinations.

## 2 Maximum Value Vertex Guard is APX-complete

Suppose a polygon $P$ without holes is given with weighted disjoint line segments on its boundary. Our line segments are open intervals $(a, b)$. The goal is to place $k$ vertex guards maximizing the weight of the overseen boundary. The formal definition follows:

**Definition 1** *Given is a polygon $P$ without holes and an integer $k > 0$. Assume the boundary of $P$ is subdivided into disjoint line segments with non negative weights (see figure 1). The goal of the* Maximum Value Vertex Guard *problem is to place $k$ vertex guards so that the total weight of the set of line segments overseen is maximum.*

We will prove that Maximum Value Vertex Guard is APX-hard. We propose a reduction from Max-5-occurrence-3-Sat problem (known to be APX-hard [10]) and we show that it is a gap preserving reduction. Let us recall the formal definition of the Max-5-occurrence-3-Sat problem:

**Definition 2** *Let $\Phi$ be a boolean formula given in conjuctive normal form, with each clause consisting of at most 3 literals and with each variable appearing in at most 5 clauses. The goal of* Max-5-occurrence-3-Sat *problem is to find a truth assignment for the variables of $\Phi$ such that the number of satisfied clauses is maximum.*

### 2.1 Construction part of the reduction

For every literal, clause and variable of the given boolean expression, we construct a corresponding pattern as shown in figure 2. Figure 2a shows a clause pattern with 3 literal "ear" patterns. It is possible to oversee the whole literal pattern with one vertex guard *only* if she is placed on vertex $F_{lit}$ or $T_{lit}$. Figure 2b shows a variable pattern with two "legs" and a "tail". Variable patterns are augmented with additional spikes described below (see figure 3). Finally we add an "ear" pattern in the upper left corner of the polygon and the construction is

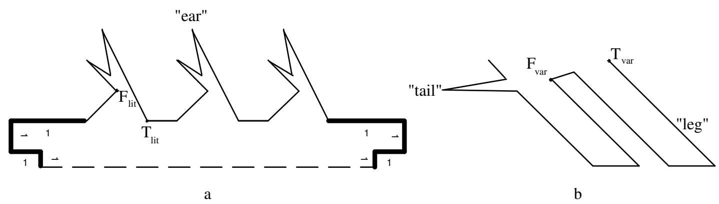


**Fig. 2.** a) a clause pattern with 3 literal patterns, b) a variable pattern

complete. A guard on vertex $w$ oversees both “legs” of every variable pattern. An example is shown in figure 4.

For every occurrence of a literal in the boolean expression, i.e. for every literal “ear” pattern, we add two “spikes” to the corresponding variable pattern: if it is a positive (negative) literal, we add the two spikes as shown in figure 3a (3b). The spike which is overseen by vertex $F_{lit}$ ($T_{lit}$) is called FALSE (TRUE) spike. Notice in figure 3 that the base of the FALSE spike is the line segment $(a, F_{lit})$, whereas the base of the TRUE spike is $(T_{lit}, b)$ and not $(T_{lit}, c)$. The purpose of this is that no vertex of the clause side (see figure 4) can oversee more than one spike (in the variable side).

Three guards are necessary and sufficient in order to oversee a literal “ear”, its corresponding variable pattern (two “legs” and a “tail”) and its corresponding spikes. One of them is placed on vertex $w$ and oversees the “legs” of the variable pattern. The other two are placed on vertices: i) $\{F_{var}, F_{lit}\}$, or $\{T_{var}, T_{lit}\}$, for positive literals, or ii) $\{F_{var}, T_{lit}\}$, or $\{T_{var}, F_{lit}\}$, for negative literals. We assign value 8 to every edge of the polygon, except the “cheap” edges of the clause patterns depicted in figure 2a, to which we assign value 1. We set the number of available guards $k = l + n + 1$, where $l$ is the number of occurrences of literals and $n$ is the number of variables of the boolean expression.

### 2.2 Transformation of a feasible solution

Suppose a truth assignment for the boolean expression is given. We will construct a guard placement that corresponds to the given truth assignment. We place $k = l + n + 1$ guards on vertices of the polygon that we constructed in section 2.1, as follows: We place in each variable pattern a guard on vertex $F_{var}$ ($T_{var}$), if the truth value of the corresponding variable is FALSE (TRUE). We place in each literal pattern a guard on vertex $F_{lit}$ ($T_{lit}$), if the truth evaluation of the literal is FALSE (TRUE). Finally we place a guard in the additional “ear” pattern, on vertex $w$. Thus, every literal pattern is overseen. Furthermore, every variable pattern is overseen by guards placed as described. The “legs” of variable patterns are overseen by the guard on vertex $w$.

Conversely, given a placement of $l + n + 1$ guards on the resulting polygon which is an instance of Maximum Value Vertex Guard we will construct a corresponding truth assignment for the original Max-5-occurrence-

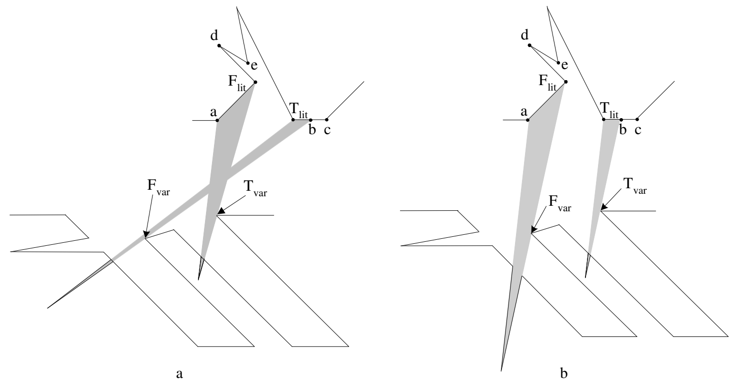


**Fig. 3.** a) two spikes corresponding to an occurrence of a positive literal in a clause. Both spikes and the "ear" are overseen by two guards placed, e.g., on $F_{var}$ (oversees left spike and "tail") and on $F_{lit}$ (oversees right spike and "ear"). b) two spikes corresponding to an occurrence of a negative literal in a clause. Both spikes and the "ear" are overseen by two guards placed, e.g., on $T_{var}$ (oversees right spike and "tail") and on $F_{lit}$ (oversees left spike and "ear").

3-Sat instance. First we modify the placement of guards by placing a) only one guard in every variable pattern on one of the vertices $F_{var}$ or $T_{var}$, b) only one guard in every literal pattern on vertex $F_{lit}$ ($T_{lit}$) if the corresponding TRUE (FALSE) spike of the variable pattern is overseen by its guard, c) one guard in the additional "ear" pattern on vertex $w$. In more details: given a placement of $k = l + n + 1$ guards with a total overseen boundary value $B$, we will modify the guard placement so that the total value overseen is $\geq B$, and so that with the exception of some "cheap" edges with weight 1, the modified guard placement achieves: a) full overseeing of all polygon edges and b) "consistent" placement on two vertices out of the four $F_{lit}, T_{lit}, F_{var}, T_{var}$ for all literals. Guard placement follows:

i) We place one guard on vertex $w$ of the additional "ear" pattern.

ii) For every variable pattern: a) If there is only one guard in the pattern placed on a vertex which oversees a spike, we place her on $F_{var}$ ($T_{var}$) if $F_{var}$ ($T_{var}$) oversees the same spike. b) In all other cases (no guards, one guard overseeing no spikes, at least two guards) we place one guard on $F_{var}$ ($T_{var}$) if $F_{var}$ ($T_{var}$) oversees more FALSE spikes than $T_{var}$ ($F_{var}$).

iii) For every literal we place one guard on $F_{lit}$ ($T_{lit}$) if the corresponding FALSE (TRUE) spike of the variable pattern is not overseen by the guard placed in the variable pattern.

We will prove in section 2.3 (see Lemma 2) that the total value overseen is at least $B$.

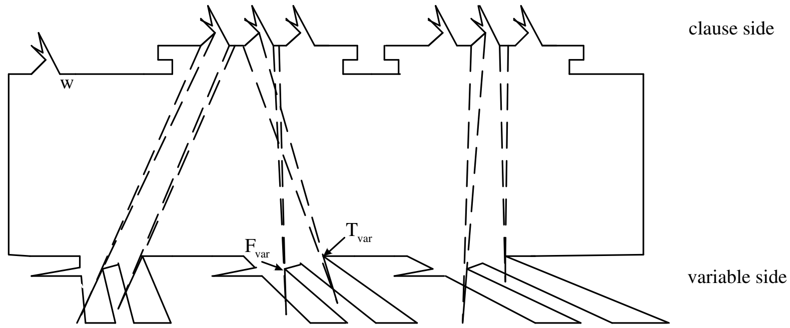


**Fig. 4.** resulting polygon

Now we can construct a truth assignment as follows: assign TRUE (FALSE) to a variable if the corresponding variable pattern has a guard on vertex $T_{var}$ ($F_{var}$).

### 2.3 Analysis of the reduction

Let $I$ be an instance of Max-5-occurrence-3-Sat with $n$ variables, $l$ occurrences of literals and $m$ clauses ($l \leq 3m$). Let $I'$ be the instance of Maximum Value Vertex Guard (constructed as in 2.1) with $k = l + n + 1$. Let $M$ be the total value of the boundary.

**Lemma 1.** *If $OPT(I) = m$ then $OPT(I') = M$.*

*Proof.* Suppose there exists a truth assignment such that all $m$ clauses are satisfied. If we place $l + n + 1$ guards in the polygon as in 2.2, then it is easy to see that the whole boundary of the polygon is overseen. So the total value of overseen edges is $M$.

Note that Lemma 1 is true no matter what the values of the cheap edges are. However we must carefully choose the values of cheap edges in order to prove Lemma 2. We want to find an optimal placement of guards in which for many clause patterns at least one of the $T_{lit}$ vertices is occupied by a guard. Thus the values of cheap edges should not be 0. We also want to cover all non-cheap edges possibly leaving some cheap ones uncovered. For every false clause of the boolean formula cheap edges in the corresponding clause pattern will be left uncovered.

**Lemma 2.** *If $OPT(I') \geq M - 8\epsilon m$ then $OPT(I) \geq m(1 - \epsilon)$.*

*Proof.* Suppose there exists an $\epsilon > 0$ and a placement of the $l + n + 1$ guards in $I'$ so that the total value of overseen boundary is at least $M - 8\epsilon m$. After the modification of guard placement described in 2.2, $k = l + n + 1$ guards oversee the whole boundary except possibly some "cheap" edges and the total value overseen is at least $M - 8\epsilon m$: Notice that if we place a guard on vertex $F_{lit}$ or $T_{lit}$ of an "ear" which has no guards we certainly increase the overseen value by at least 16, because edges $(F_{lit}, d)$ and $(d, e)$ can not be overseen by any outside guard. Similarly a guard placed on $F_{var}$ or $T_{var}$ of a variable pattern that has no guards, certainly increases the overseen value by at least 16 (namely weight of the two "tail" edges).

We will discuss two cases pertaining to guard placement in "ears":

a) The original placement had two guards on vertices $T_{lit}$ and $F_{lit}$ of a literal "ear" pattern and after the modification, one guard was placed on vertex $F_{lit}$ of the pattern. The total value may have been decreased by at most 8 (because "cheap" edges may now be missed) but it is increased by at least 16 (because the free guard was placed in an unguarded pattern).

b) The original placement had one guard on vertex $T_{lit}$ of a literal "ear" pattern and after the modification she was moved to vertex $F_{lit}$ of the pattern:

i) If the corresponding FALSE spike was not overseen in the original placement (by a guard in the variable pattern), the total value may have been decreased by at most 8 (because "cheap" edges may now be missed) but it is increased by at least 16 (because the FALSE spike is now overseen by the guard on $F_{lit}$).

ii) If the corresponding FALSE spike was overseen in the original placement (by a guard $g$ in the variable pattern), then it must be the case that the variable pattern had originally at least two guards and after the modification, guard $g$ was removed and placed in another pattern because there was another guard that was overseeing the most FALSE spikes in the variable pattern. The guard $g$ was overseeing at most 2 FALSE spikes because a variable pattern has at most 5 FALSE spikes, since a variable appears in at most 5 clauses of the boolean formula. Thus, for every variable pattern, guards have been moved from vertex $T_{lit}$ to $F_{lit}$ in at most two literal patterns. The total value may have been decreased by at most 16 (because "cheap" edges of two clauses may now be missed) but it is increased by at least 16 (because at least one free guard was placed in an unguarded pattern).

We can now construct a truth assignment for $I$ as in 2.2 that leaves at most $\epsilon m$ clauses unsatisfied that correspond to $\epsilon m$ clause patterns not overseen by any guard in $I'$.

From Lemma 1 and the contraposition of Lemma 2 the following theorem holds:

**Theorem 1** *Let $I$ be an instance of* Max-5-occurrence-3-Sat *problem with $n$ variables, $m$ clauses and $l \leq 3m$ occurrences of literals. Let $I'$ be the instance of* Maximum Value Vertex Guard *problem (constructed as in 2.1) with $k = l + n + 1$. Let $M$ be the total value of the boundary of the polygon. Then:*

- $OPT(I) = m \rightarrow OPT(I') = M$
- $OPT(I) \leq m(1-\epsilon) \rightarrow OPT(I') \leq M - 8\epsilon m$

Thus our reduction is gap-preserving [10].

In [9, 10] it was proved that the Max-5-occurrence-3-Sat problem with parameters $m$ and $(1-\epsilon)m$ for some $\epsilon > 0$, where $m$ denotes the number of clauses in instance $I$, is NP-hard to decide.

Therefore, we obtain that unless $P = NP$, no polynomial time approximation algorithm for Maximum Value Vertex Guard can achieve an approximation ratio of $\frac{M}{M-8\epsilon m}$.

Considered that $M = nV + lL + 2lS + mC + E$ where $V$ denotes the total value of a variable pattern ("legs", "tail", "leg-edges" between spikes, plus one edge that links the variable pattern with the next one on the right : $104 \leq V \leq 168$), $L$ denotes the total value of a literal pattern ("ear" : $L = 40$), $S$ denotes the total value of a spike pattern ($S = 16$), $C$ denotes the total value of a clause pattern without "ears" plus one edge that links the clause pattern with the next one on the right ($16 \leq C \leq 32$) and $E$ denotes the total value of the additional ear pattern and the remaining edges of the polygon ($E = 80$), then:

$$M \leq 3mV + 3mL + 6mS + mC + E$$

With a few calculations it turns out:

$$\frac{M}{M - 8\epsilon m} = \frac{1}{1 - \frac{8\epsilon m}{M}} \geq \frac{1}{1 - \frac{8\epsilon}{3V+3L+6S+C+E}} \geq 1 + \epsilon'$$

for some $\epsilon'$ that depends on $\epsilon$. Therefore:

**Theorem 2** Maximum Value Vertex Guard *is APX-hard.*

On the other hand the Maximum Value Vertex Guard problem can be approximated within a constant ([12]). Therefore:

**Corollary 1** Maximum Value Vertex Guard *is APX-complete.*

## 3 A bunch of APX-complete Art Gallery problems

In this section we propose appropriate modifications of the reduction of section 2 in order to show APX-hardness for a number of variations of Maximum Value Vertex Guard. We also give constant ratio approximation algorithms for these problems (where not already known), thus showing them to be APX-complete.

The case in which guards are placed on edges (guards occupying whole edges), is called Maximum Value Edge Guard problem. A guard which is occupying a whole edge, can be thought of as a mobile guard able to move on the edge.

**Proposition 1** Maximum Value Edge Guard *is APX-hard.*

*Proof.* We show the result by a reduction from Max-5-occurrence-3-Sat to Maximum Value Edge Guard. The reduction follows the one in section 2 using modified literal and variable patterns, as shown in figure 5. It is not hard to check that the properties mentioned in Theorem 1 hold here as well.

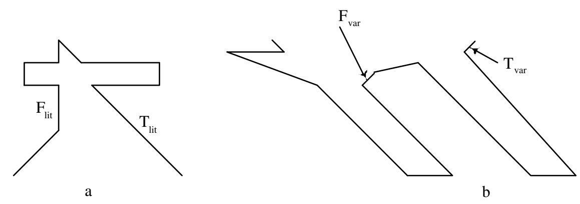


**Fig. 5.** a) a literal pattern and b) a variable pattern for edge guard problems

For the rest of our problems we consider both vertex-guard and edge-guard versions and we use the corresponding construction for our reductions, i.e. the one of section 2.1 for the vertex-guard problems and the one used in Proposition 1 (with the modified literal and variable patterns) for the edge-guard problems. All the reductions are from Max-5-occurrence-3-Sat to the problem in hand.

Now we will relax the meaning of guarding: "watching a valuable painting", i.e. "overseeing a part of it" instead of "overseeing all of it".

**Proposition 2** *The watching versions of* Maximum Value Vertex Guard *and* Maximum Value Edge Guard *problems are APX-hard.*

*Proof.* Let us describe a reduction from Max-5-occurrence-3-Sat to the watching version of Maximum Value Vertex/Edge Guard. We first construct the polygon using the appropriate gadgets (depending on the kind of guards as explained above). We then discretize the boundary using the Finest Visibility Segmentation (FVS) described in [12]. Let us recall this technique: we use the visibility graph $V_G(P)$. By extending edges of $V_G(P)$ inside $P$ up to the boundary of $P$ we obtain a set of points $FVS$ on the boundary of $P$ ($FVS$ includes of course all corners of $P$) (see Figure 6a). There are $O(n^2)$ points in $FVS$ and these points are endpoints of line segments with the following property: for any vertex $y$, a segment $(a, b)$ defined by consecutive $FVS$ points is visible by $y$ iff it is watched by $y$. Furthermore $(a, b)$ is watched (and visible) by an edge $e$ iff it is watched by any point in $FVS \cap e$. Thus we can find the set of line segments $E'(v)$ ($E'(e)$) which are watched by a vertex $v$ (edge $e$) within polynomial time.

Every edge in a clause pattern will be subdivided into $O(n)$ FVS segments, because it can be watched only by vertices in variable patterns. Let $\delta > 0$ be an integer such that the number of FVS segments in any of the (previously) "cheap" edges of a clause pattern is at most $\delta n$. We assign value 1 to every FVS segment which belongs to a (previously) "cheap" edge of a clause pattern. We assign value $8\delta n$ to every other segment. The properties of Theorem 1 hold (details are omitted for brevity).

Consider now the following problem:

**Definition 3** *Given is a polygon $P$ without holes and an integer $k > 0$. Let $L(b)$ be the euclidean length of the line segment $b$. The* Maximum Length

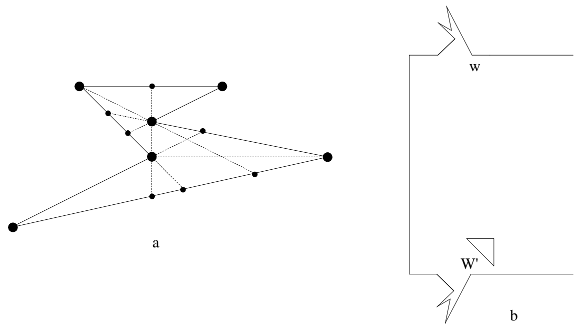


**Fig. 6.** a) subdividing the boundary into line segments with endpoints in $FVS$, b) the left part of the polygon with the hole

VERTEX/EDGE GUARD *problem asks to place k vertex (edge) guards so that the euclidean length of the* overseen *part of P's boundary is maximum.*

**Proposition 3** MAXIMUM LENGTH VERTEX/EDGE GUARD *is APX-hard.*

*Proof.* For the construction part of the reduction, we construct the polygon using the gadgets for vertex-guard or edge-guard version with the following additional modification: we make sure that the length of every (previously) "cheap" edge in a clause pattern is designed at least 8 times shorter than any other edge of the polygon. Now the properties of Theorem 1 hold here as well (again details are omitted).

All these problems may also appear in polygons with holes. Holes in polygons are useful because they give us the chance to model reality better (holes represent obstacles) and to place guards in the interior of the polygon (on predefined places), on vertices or edges of the holes. We remind the reader that for MINIMUM VERTEX/EDGE GUARD for polygons with holes, no polynomial time approximation algorithm can guarantee an approximation ratio of $\frac{1-\epsilon}{12}\ln n$ for any $\epsilon > 0$, unless $NP \subseteq TIME(n^{O(\log\log n)})$ ([7, 5]).

**Proposition 4** *The following problems are all APX-hard for polygons with holes.*

- *The* **overseeing** *version of* MAXIMUM VALUE VERTEX/EDGE GUARD
- *The* **watching** *version of* MAXIMUM VALUE VERTEX/EDGE GUARD
- MAXIMUM LENGTH VERTEX/EDGE GUARD

*Proof.* In the construction part of the corresponding reduction for every one of the above problems we add a hole and another "ear" pattern in the left lower corner of the polygon as shown in figure 6b. Theorem 1 again holds.

For the problems of Propositions 1-4, polynomial time constant ratio approximation algorithms are presented in [12]. Hence:

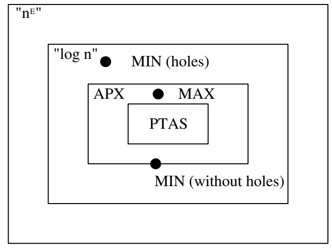


**Fig. 7.** Classifying Art Gallery problems in approximation classes. We use: "$n^\epsilon$" to denote the class of problems with $O(n^\epsilon)$ approximation ratio, "$\log n$" for the class of problems with $O(\log n)$ approximation ratio, APX for the class of problems with constant approximation ratio and PTAS for the class of problems with an infinitely close to 1 constant approximation ratio.

**Theorem 3** *The following problems are all APX-complete for polygons with or without holes.*

- *the* **overseeing** *version of* Maximum Value Vertex/Edge Guard
- *The* **watching** *version of* Maximum Value Vertex/Edge Guard
- Maximum Length Vertex/Edge Guard

Another variation of the Maximum Value Vertex/Edge Guard problem is the maximization of the total value of overseen valuable paintings where only the dimensions of the paintings are given. So the goal is to place vertex/edge guards as well as to place the given paintings on the boundary of the polygon. The problem called Maximum Value Vertex/Edge Guard PP is also APX-complete ([16]).

## 4 Conclusions

We have proved that overseeing a maximum value part of a weighted boundary of a polygon without holes, using at most $k$ vertex guards (Maximum Value Vertex Guard) is APX-complete. We have also proved that the variations involving i) edge guards (Maximum Value Edge Guard), ii) polygons with holes and iii) watching instead of overseeing the boundary, are APX-complete. In addition, we have shown that Maximum Length Vertex Guard and Maximum Length Edge Guard for polygons with or without holes are APX-complete. We end up with a hierarchy of Art Gallery problems which is shown in figure 7.

Maximization Art Gallery problems for polygons with or without holes that we studied (*MAX* in figure 7) are APX-hard while at the same time they have constant approximation ratios (thus APX-complete). Minimization Art Gallery problems for: a) polygons with holes (*MIN holes* in figure 7) are $\log n$-hard and have $O(\log n)$ approximation ratios (thus $\log n$-complete), b) polygons without holes (*MIN without holes* in figure 7) are APX-hard and have $O(\log n)$ approximation ratios but it is not known whether they have constant approximation ratios or whether they are $\log n$-hard.

We have shown that our gap-preserving reduction can be applied with minor modifications to a number of problems. New elements of problems studied here are: a) **weighted line segments** of the polygon's boundary and b) the useful and promising concept of **watching** line segments as opposed to completely **overseeing** them. Interesting open problems arise if we consider all the above problems in the case where exhibits may lie in the interior of the polygon.

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