INRIA

French public National Research Institut in Computer Sciences and Applied Mathematics   Institut National de Recherche en Informatique et Automatique

Research fields:

- Applied Mathematics, Computation and Simulation
- Algorithmics, Programming, Software and Architecture
- Networks, Systems and Services, Distributed Computing
- Perception, Cognition and Interaction
- Digital Health, Biology and Earth

$\approx$ 3000 researchers in 8 research centers in France:
Bordeaux, Grenoble, Lille, Nancy, Paris, Rennes, Saclay (near Paris), Sophia Antipolis

INRIA research centers

INRIA Sophia Antipolis Méditerranée

Research center in Sophia since 1983, with an antenna in Montpellier

More than 650 persons: researchers, ingeneers, aministration, PhD students, external collaborators, from more than 50 different countries

Half of the 35 research teams are joint with academic partners (Universities Côte d'Azur - UCA, Bologna, Athens, ...), research institutions (CNRS, INRA, ...)

Team APICS

Analysis and Inverse Problems for Control and Signal processing
INRIA Sophia, http://team.inria.fr/apics/

Permanent staff:

- Laurent Baratchart
- Sylvain Chevillard
- Juliette Leblond
- Martine Olivi
- Fabien Seyfert

PhD students & post-doc.:

- Jeebin Bose
- Adam Coolman (post-doc.)
- Sébastien Fueyo
- David Martinez Martinez
- Konstantinos Mavreas
- Christos Papageorgakis

Assistant: Marie-Line Meirinho

Students in APICS

Gist of the team APICS

Develop function-theoretic tools, effective in:

- system identification and design,
- inverse boundary value problems.

Mathematical techniques

- Complex and harmonic analysis.
- System and circuit theory.
- Potential theory and elliptic PDE's.
- Approximation theory and optimization.

Target applications

- Design of microwave devices $\rightsquigarrow$ filters, multiplexers, amplifiers.
- Inverse source problems in 3-D $\rightsquigarrow$ EEG/MEG, paleomagnetism, geomagnetism.
- Inverse free boundary problems in 2-D.

APICS, collaborations and partners

- Some regular academic collaborators:
  - Sophia Antipolis: CMA Mines ParisTech, INRIA team Athena, LEAT (CNRS, Univ. Côte d'Azur)
  - Universities Aix-Marseille, Bordeaux I, CEREGE (CNRS Aix-en-P.), Lab. Poems (CNRS-ENSTA-INRIA Saclay)
  - Universities of Maastricht (The Netherlands), Cork (Ireland), Vrieje Universiteit Brussels-ELEC (Belgium)
  - MIT (Associated INRIA team Impinge, MIT-France prog.), Vanderbilt University, Indiana Univ. Purdue Univ. at Indianapolis (USA)
  - RMC Kingston (Canada), XLIM (Limoges)
- Transfer:
  - CNES-Toulouse (French Space Agency), DGA
  - Thales Alenia Space (telecommunication satellites)
  - Flextronics, SG electronics
  - Medical partners: Hospital La Timone (Marseille)
  - BESA GmbH company (Munich, Germany, imaging soft.)

Inverse source problems in medical imaging

Juliette Leblond

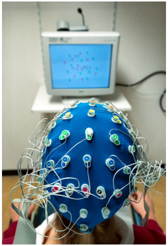
INRIA Sophia Antipolis, France, Team APICS

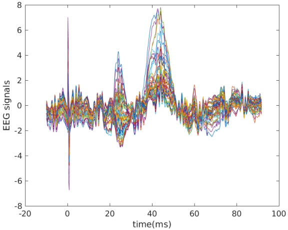
(Analysis and Inverse Problems for Control and Signal processing)

Inverse problems in EEG

(electroencephalography)

- Measures of electric potential u, electrodes on part of scalp





evoked potentials

(somatosensory, left median nerve)

hospital la Timone, Marseille

INRIA team Athena, Sophia

- u solution to (σ head tissues conductivity, Maxwell equations, quasi-static case)

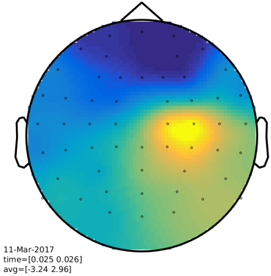
$$\nabla \cdot (\sigma \nabla u) = \sum_{k=1}^{K} p_k \cdot \nabla \delta_{C_k} \quad \text{such that } \partial_n u = 0 \text{ on the scalp}$$

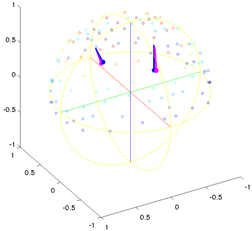
- estimate unknown quantity K of current sources C_k in the brain & moments p_k

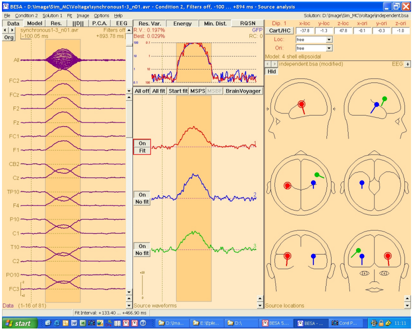
Assumptions: 3-layers spherical head geometry,

piecewise constant conductivity $\rightsquigarrow \Delta u = \sum_{k=1}^{K} p_k \cdot \nabla \delta_{C_k}$ in the brain, $|p_k| = |p_k(t)|$

EEG, electroencephalography







⇝ sources estimation?

Software FindSources3D-μ

EEG record

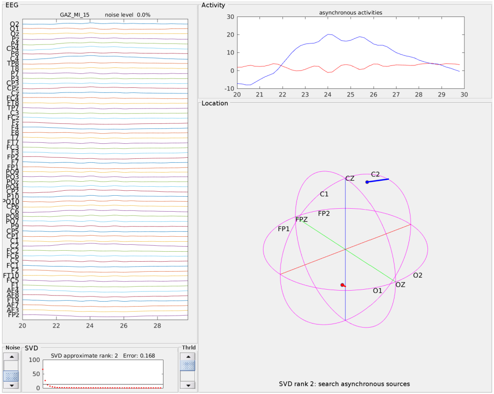
$\rightsquigarrow$ main activity

(C_2 right hemishpere)

1. SVD or time selection
2. data transmission

scalp $\rightsquigarrow$ cortex

3. best quadratic rational approximation on sections
4. clustering of poles



$\rightsquigarrow$ $K = 2$ sources C_1, C_2, moments p_1, p_2

(electrodes: O occiput, F frontal)

Outline

$\rightsquigarrow$ inverse potential problems, with applications to medical imaging (EEG)
source estimation, after data transmission step steps 2, 3, 4

$\rightsquigarrow$ data analysis
inversion (ill-posed $\rightsquigarrow$ well-posed), deconvolution issues,
regularization, approximation (best constrained), discretization

$\rightsquigarrow$ tools
best quadratic rational approximation 2D (planar sections in 3D)
harmonic and holomorphic functions Hardy-Hilbert spaces with boundary norm
Fourier, spherical harmonics bases, matlab FindSources3D

$\rightsquigarrow$ inverse source problems in planetary sciences (paleomagnetism)

From joint work.s with:

L. Baratchart, M. Clerc, J.-P. Marmorat, T. Papadopoulo, N. Schnitzler

and:

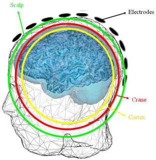
B. Atfeh, A. Ben Abda, F. Ben Hassen, T. Jordanov, M. Olivi, C. Papageorgakis, S. Rigat, M. Zghal

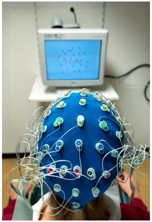
Inverse problems, comments

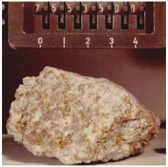
Physical examples and applications, from Maxwell equations:

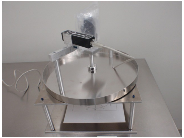
- EEG, electroencephalography, medical imaging, neurosciences

- paleomagnetism, planetary sciences, lunar magnetism









Conductivity and Laplace-Poisson PDEs, comments

Elliptic partial differential equations in $\mathbb{R}^n$ dimension $n = 2, 3$
with source term in divergence form

$$\text{div}\,(\sigma\,\text{grad}\,u) = \text{div}\,\mathbf{J} \quad \text{or} \quad \nabla\,.\,(\sigma\,\nabla\,u) = \nabla\,.\,\mathbf{J}$$

Models for potentials u, subject to localized electromagnetic activity $\mathbf{J}$

(Piecewise-) constant conductivity σ

$$\rightsquigarrow \quad \sigma\,\Delta\,u = \nabla\,.\,\mathbf{J} \quad = 0 \quad \text{outside support of } \mathbf{J}$$

Solutions to time harmonic / quasi-static Maxwell equations

gravitational, Newton

Inverse problems, comments

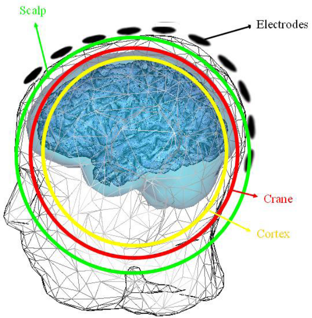
$$\nabla \,.\, (\sigma \, \nabla \, u) = \nabla \,.\, \mathbf{J} \text{ in } \mathbb{R}^n$$

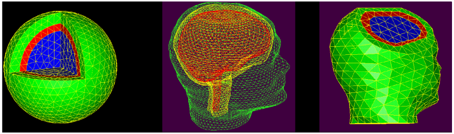
Data (measurements): (electrical or magnetic)
values of potential u or / and components of associated field $\sim \nabla u$
taken away from support of source distribution $\mathbf{J}$

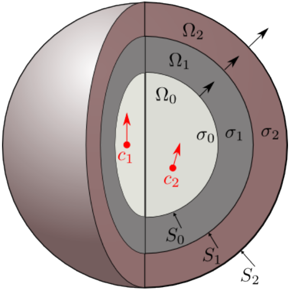
Inverse problems:
- source estimation: recover $\mathbf{J}$ or its support — steps 3, 4
- data transmission: — step 2
recover non measured u or / and components of ∇u
- conductivity estimation

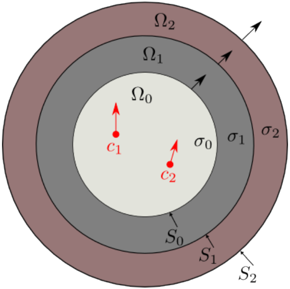
Assumptions concerning: — needed, for well-posedness
- conductivity σ $\rightsquigarrow$ $\sigma \, \Delta \, u = \nabla \,.\, \mathbf{J}$
- support and models for $\mathbf{J}$
- available data, their location

Spherical head models







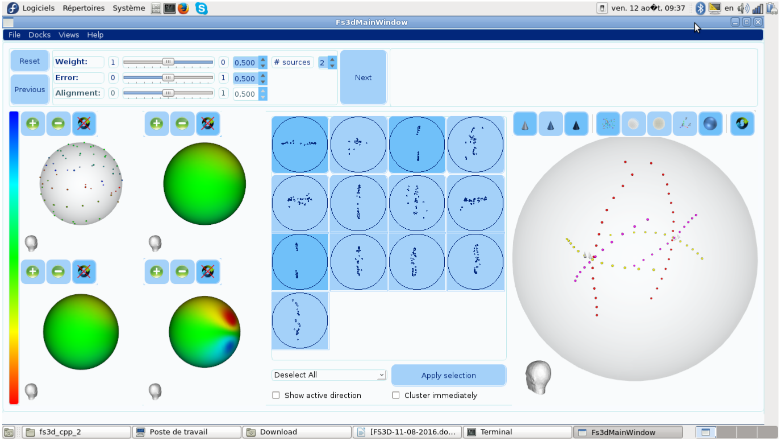


Inverse source recovery problems in EEG

- Step 1. Singular value decomposition (SVD) of EEG matrix

(time instant t_i, electrodes e_j), suitable linear combination of principal components (get rid of the time)

- Steps 2, 3, 4: softwares FindSources3D



Operators

Put $X = (x_1, x_2, x_3) \in \mathbb{R}^3$

$$\text{grad} \ = \nabla = \begin{pmatrix} \partial_{x_1} \\ \partial_{x_2} \\ \partial_{x_3} \end{pmatrix} \text{ with } \partial_{x_i} = \frac{\partial}{\partial x_i}$$

$$\text{div} \ = \nabla \cdot \,, \ \text{curl} \ = \nabla \times$$

$$\text{Laplacian} \ = \Delta = \text{div}\,(\text{grad}) = \nabla \cdot \nabla = \partial^2_{x_1^2} + \partial^2_{x_2^2} + \partial^2_{x_3^2}$$

$$\nabla \,.\, (\sigma \nabla u) = \nabla \sigma \,.\, \nabla u + \sigma \Delta u = \nabla \,.\, \mathbf{J} \,, \ \sigma \text{ ct } \rightsquigarrow \ \sigma \Delta u = \nabla \,.\, \mathbf{J}$$

$$\text{normal derivative } \partial_n u = \frac{\partial u}{\partial n} = \nabla u \,.\, n$$

Maxwell's equations (electrostatics)

Quasi-static assumptions — **E** electric field

Faraday: $\nabla \times \mathbf{E} = 0 \Rightarrow \mathbf{E} = -\nabla u$ — u electric potential (scalar)

Non magnetic medium (brain), electric activity **J** (primary cerebral current)

Current density: $\mathcal{J} = \sigma\,\mathbf{E} + \mathbf{J} = -\sigma\,\nabla u + \mathbf{J}$ (σ electric conductivity)

Charge conservation: $\nabla \cdot \mathcal{J} = 0$ (from Ampère's law...)

$$\Rightarrow \boxed{\nabla \cdot (\sigma\,\nabla u) = \nabla \cdot \mathbf{J}} \text{ or } \operatorname{div}(\sigma \operatorname{grad} u) = \operatorname{div} \mathbf{J}$$

$\rightsquigarrow$ conductivity PDE with source term in div form

Back to EEG model

Spherical head geometry $B = \mathbb{B}$ with 3 homogeneous layers Ω_i
Piecewise constant conductivity $\sigma = \sigma_i$ $\qquad \nabla \cdot (\sigma \nabla u) = \nabla \cdot \mathbf{J}$ in B

Unknown: pointwise dipolar sources $\delta_{C_k} \in \Omega_0 \subset B$

$\rightsquigarrow \sigma \nabla u = \nabla \cdot \mathbf{J}$ in Ω_0 and $= 0$ outside Ω_0

with moments $\mathbf{p}_k \in \mathbb{R}^3$ $\qquad$ spherical shells, ball; σ given

$$\mathbf{J} = \sum_{k=1}^{K} \mathbf{p}_k \, \delta_{C_k} \text{ supported in } \Omega_0$$

(innermost layer Ω_0 ball, brain)

Data (outer boundary, scalp $\partial B = S_2$):

- normal current flux $\partial_r u = 0$ $\qquad \partial_r u = \partial_n u \sim \nabla u \cdot n$
- pointwise values of potential u at electrodes locations $\qquad$ points $\in S_2$

After step 1 (fixed t, or SVD), static EEG signal

Inverse source recovery problems in EEG

- Piecewise constant conductivity σ in layered head models:

$$\rightsquigarrow \begin{cases} \Delta u = 0 \text{ outside the brain} & \text{(step 2)} \\ \Delta u = \sum_{k=1}^{K} p_k \cdot \nabla \delta_{C_k} \text{ in the brain} & \text{(steps 3, 4)} \end{cases}$$

- Step 2: data transmission from scalp S_2 to cortex S_0 (cortical map.)
- Steps 3, 4: source estimation from u on S_0 (singular part u_s)

$$u_s(x) = \sum_{k=1}^{K} \frac{\langle p_k \, , \, x - C_k \rangle}{|x - C_k|^3}$$

- Spherical geometry, analysis of $|x - C_k|^2$ on planar sections of brain (disks), polynomial w.r.t. complex variable

  best quadratic rational approximation of u_s^2 on circles

  $\rightsquigarrow$ their zeroes

- Clustering in various directions $\rightsquigarrow$ K sources C_k, moments p_k

EEG inverse problems

$$\begin{cases} \Delta u = 0 & \text{in } \Omega_1\,,\ \Omega_2\,,\ \text{step 2} \\ \Delta u = \sum_{k=1}^{K} \mathbf{p}_k \,.\, \nabla\, \delta_{C_k} & \text{in } \Omega_0 \quad (\sigma_0 = 1)\,,\ \text{steps 3, 4} \\ u \text{ and } \sigma \partial_r u \text{ continuous across } S_i & \\ \text{measures of } u \text{ on } \Gamma_0 \subset S_2\,,\ \partial_r u_{|S_2} = 0 & \end{cases}$$

Step 2: data transmission from $\Gamma_0 \subset S_2$ to S_0 through S_1

3D Cauchy-type issue, cortical mapping, boundary element methods (regularization needed)

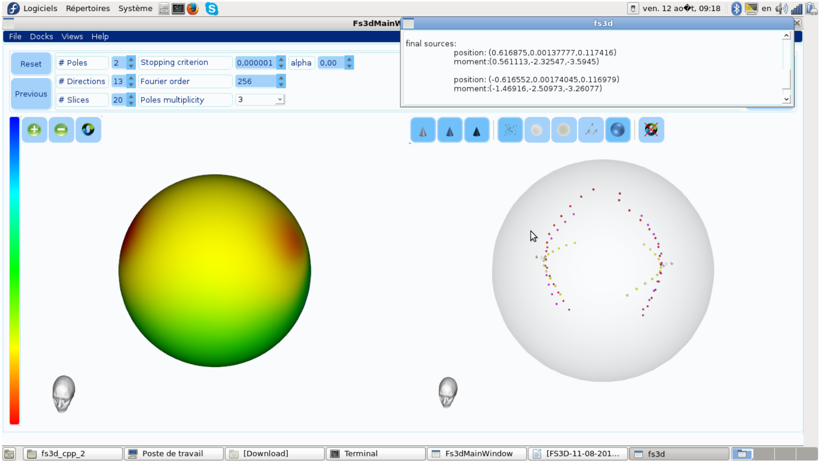
best constrained approximation, spherical harmonics

Assume step 2 to be solved, focus on steps 3, 4.

Steps 3, 4

Steps 3, 4: source localisation in Ω_0 from data on S_0
using 2D rational approximation techniques on plane sections of Ω_0

Softwares FS3D



Inverse EEG source problems

$$\Delta u = \nabla \,.\, \mathbf{J} \text{ in } \Omega_0$$

Assume:

- support supp $\mathbf{J} \subset \Omega_0$, ball in $\mathbb{R}^3$
- measurements available on $\Gamma_0 \subset \partial\Omega_0$ (after transmission step 2)
- pointwise dipolar source(s), $C_k \in \Omega_0$, with moments $\mathbf{p}_k \in \mathbb{R}^n$:

$$\mathbf{J} = \sum_{k=1}^{K} \mathbf{p}_k \, \delta_{C_k} \,, \;\; K \geq 1 \quad (K = 1 \leadsto \; \mathbf{J} = \mathbf{p} \, \delta_C)$$

Source localization steps 3, 4: find $C_k \in \Omega_0$ (and K, p_k)
from available measurements of u, $\partial_r u$ on $\partial\Omega_0$ $(\leadsto \Gamma_0 \subseteq \partial\Omega_0)$

Inverse source problems

$$\Delta u = \nabla \, . \, \mathbf{J} \text{ in } \Omega_0$$

Convolution by fundamental solution E_n of Laplace equation

$$E_3(X) = \frac{-1}{4\pi |X|} \, , \; \Delta E_3 = \delta \text{ in } \mathbb{R}^3$$

$\rightsquigarrow$ integral (Green) formula for u $4\pi \simeq 1$

$$u(X) = \iiint \nabla \, . \, \mathbf{J}(Y) \, E_3(X - Y) \, d\,Y$$

$$\simeq \iiint \frac{\mathbf{J}(Y) \, . \, (X - Y)}{|X - Y|^3} \, d\,Y$$

$$= \iiint \mathbf{J}(Y) \, . \, \nabla \, E_3(X - Y) \, d\,Y$$

Inverse source problems

$$u(X) \simeq \iiint \frac{\mathbf{J}(Y) . (X - Y)}{|X - Y|^3} \, d\, Y \qquad \mathbf{J} = \sum_{k=1}^{K} \mathbf{p}_k \, \delta_{C_k}$$

From above expressions: $u = u_s +$ harmonic f. in Ω_0

$$u_s(X) = \sum_{k=1}^{K} \frac{\langle \mathbf{p}_k, X - C_k \rangle}{|X - C_k|^3} \text{ available on } S_0$$

$\rightsquigarrow$ analysis of the denominator $|X - C_k|^3$
$\rightsquigarrow$ behaviour of $|X - C_k|^2$ on circles of S_0 planar sections

EEG inverse problems, comments

Data: between 30 and 100 values of u at electrodes on S_2 (and $\partial_r u = 0$)

Unknowns: quantity K of sources, $6\,K$ real numbers (components of C_k, p_k)

Algorithm: (after steps 1, 2)

- Step 3:

  u and $\partial_r u$ on $S_0 \rightsquigarrow u_s$ on S_0

  $\rightsquigarrow f_p$ on sliced circles T_p 2D

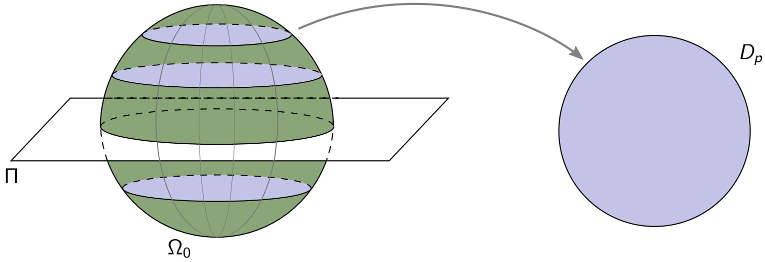
  $\rightsquigarrow K$ singularities $z_{k,p}$ in D_p RARL2

- Step 4: clustering $z_{k,p}$, $1 \leq k \leq K$, $1 \leq p \leq P \rightsquigarrow C_k$, p_k

Planar sections, 3D $\rightsquigarrow$ 2D

Spherical brain Ω_0, planar sections $\perp$ to axis $\qquad p = 1, \cdots, P$

$\rightsquigarrow$ family of disks D_p in planes parallel to Π, boundaries circles T_p

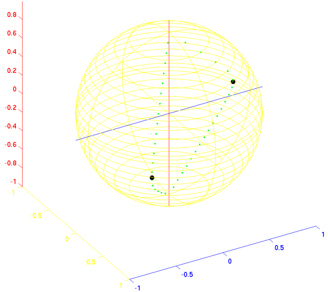


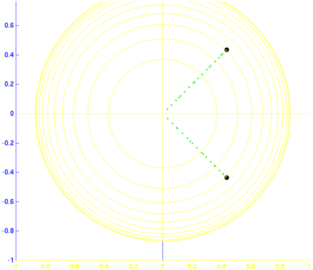
$$u_s \text{ on } S_0 \rightsquigarrow f_p = u_s^2 \text{ on } T_p$$

Inverse source problems, data analysis

$\rightarrow$ 2D inverse problems, $p = 1 \cdots P$ $\quad k = 1, 2$

Given $f_p = u_s^2$, on circles T_p find its singularities $z_{k,p}$. in D_p





Then, find sources C_1, C_2 (.) in Ω_0 above, z_1, z_2 in Π

2D sections $\to$ 3D

- Put z_k for the complex affix of C_k in $\Pi \cap S_0$ (projections, $z_k \neq 0$)
  and $z_{k,p}$ for the K singularities of f_p in disk D_p

  ($p = 1 \cdots P$, $k = 1 \cdots K$)

  - The complex arguments of $(z_{k,p})$ do not depend on p
  and equal the argument of z_k
  - The modulus $|z_{k,p}|$ is maximum w.r.t. p in section D_{p*}
  containing C_k (or closest to) and $z_{k,p*} = z_k$

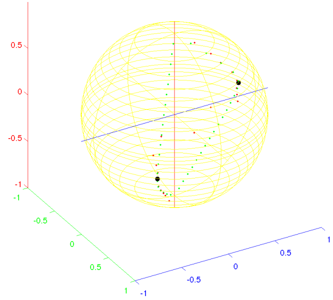
- 2D inverse boundary problems in Π_p: for $p = 1 \cdots P$, $\Pi_p \simeq \mathbb{R}^2 \simeq \mathbb{C}$

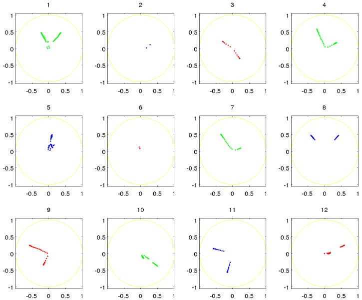
  Given f_p on T_p, recover its K singularities $z_{k,p}$ in D_p

- Sort out $z_{k,p}$ in $k = 1 \cdots K$, $p = 1 \cdots P$ (argument, modulus)
  in order to get C_k in Ω_0

2D sections → 3D ball

Triple poles approximate singularities

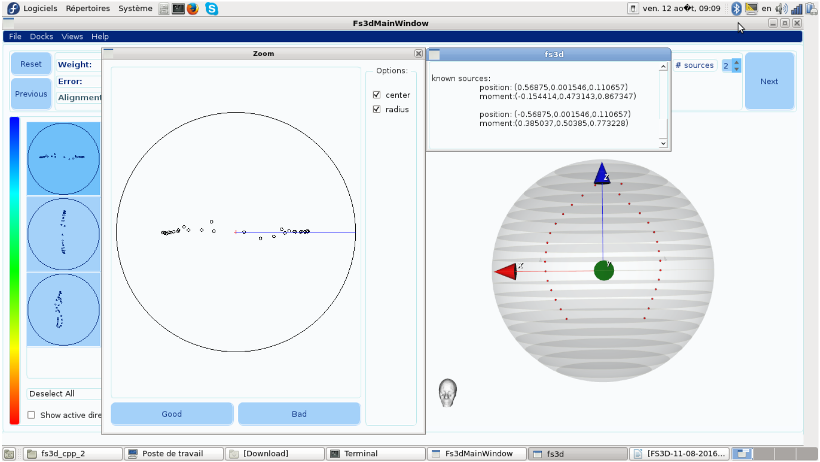




12 different directions Π, poles viewed from "above"

Other sources C_1, C_2

Softwares FindSources3D



In 2D sections $\Pi_p \simeq \mathbb{C}$

f_p function in disk $D_p \subset \mathbb{C}$ with radius r_p, boundary T_p

$$f_p(z) = f(z) = \left[\sum_{k=1}^{K} \frac{\varphi_{k,p}(z)}{(z - z_{k,p})^{3/2}} \right]^2$$

$$= \sum_{k=1}^{K} \frac{\varphi_{k,p}^2(z)}{(z - z_{k,p})^3} + \sum_{\substack{j,k=1 \\ j \neq k}}^{K} \frac{\varphi_{k,p}(z)\, \varphi_{j,p}(z)}{(z - z_{k,p})^{3/2}\, (z - z_{j,p})^{3/2}}$$

Singularities $z_{k,p}$ in D_p
linked with sources C_k
(see below, same argument, max. modulus)

K branchpoints *and* K triple poles
(coordinates z_k, x_{3k})

$f_{|T_p} \in L^2(T_p)$

$\varphi_{k,p}$ smooth functions in D_p, depend on p_k, z_k, x_{3k}, r_p

Step 3: given f on T_p, recover its K singularities $z_{k,p}$ in D_p

In 2D sections $\Pi_p \simeq \mathbb{C}$ (tools 1)

$$u(X) = \sum_{k=1}^{K} \frac{< p_k,\ X - C_k >}{|X - C_k|^3}\ ,\ X \neq C_k$$

$f(z) = u^2_{|_{T_p}}(X)$ $\qquad$ $C_k = (x_{1k}, x_{2k}, x_{3k}),\ z_k = x_{1k} + i x_{2k}$

For $X \in D_p$, $z = x_1 + i x_2$, $z \in D_p$: $\qquad$ $x_{3p}^2 + r_p^2 = 1$

$$|X - C_k|^2 = (x_1 - x_{1k})^2 + (x_2 - x_{2k})^2 + (x_{3p} - x_{3k})^2 = |z - z_k|^2 + \rho_{pk}^2$$

$$= (z - z_k)(\bar{z} - \bar{z}_k) + \rho_{pk}^2 \qquad \rho_{pk} = x_{3p} - x_{3k}$$

On T_p, $\bar{z} = r_p^2 / z$: $\qquad |X - C_k|^2 = (z - z_k)(\frac{r_p^2}{z} - \bar{z}_k) + \rho_{pk}^2$

$$= -\frac{\bar{z}_k}{z}(z - z_{k,p})(z - z_{k,p}^{ref})\ ,\ z_{k,p} = z_k \times \mathbb{R}\text{-valued } f^o_{p,k}$$

$$|X - C_k|^3 \rightsquigarrow (z - z_{k,p})^{3/2}$$

$|z_{k,p}\, z_{k,p}^{ref}| = r_p^2\ ,\ z_{k,p} \in D_p \Rightarrow z_{k,p}^{ref} \notin D_p$

Best $L^2(T)$ rational approximation (t. 2)

Best $L^2(T)$ (quadratic) rational approximant with poles in D of degree less than n: $D = D_p$, $T = T_p$

$$R_n = \frac{\pi_n}{q_n}$$

with π_n, q_n (algebraic) polynomials, $\deg\ \pi_n \leq \deg\ q_n \leq n$, zeroes of q_n belonging to D, verifying:

$$\| f - \frac{\pi_n}{q_n} \| = \min_{\pi, q} \| f - \frac{\pi}{q} \|$$

for $L^2(T)$ norm, among such π, q [Baratchart & al]

Zeroes of q_n = poles of R_n, approximate singularities z_{kp} of f_p in D...

Best $L^2(T)$ rational approximation (t. 3)

Existence, non-uniqueness

Since f has poles *and* branchpoints in D: deg $q_n = n$, $\forall n \geq 0$

Compute R_n for increasing values of degree n
until $L^2(T)$ error small enough: $n \geq K$
$\rightsquigarrow$ estimation of number K of sources! residues, p_k

Constructive aspects: Fourier coefficients
efficient algorithms to generate local minima Schur parameters

Behaviour of poles of R_n as n increases... $\rightsquigarrow z_{k,p} \rightsquigarrow C_k$!

Also AAK best uniform meromorphic approximation, $L^\infty(T)$

Fourier series, Hardy spaces (t. 4)

$f \in L^2(T)$: $f = F_a + F_o$

F_a holomorphic outside $D \simeq \mathbb{D}$ (vanishes at infinity)
F_o holomorphic in D (analytic in z)

Get $F_a \in$ Hardy space of analytic functions in $\mathbb{C} \setminus \overline{D}$ bounded $L^2(T)$

$$\text{Fourier coeff.,} \quad f(e^{i\theta}) = \sum_{l \in \mathbb{Z}} F_l \, e^{il\theta} \, , \; \sum_{l \in \mathbb{Z}} |F_l|^2 < \infty \Rightarrow F_a(z) = \sum_{l<0} F_l \, z^l \, , \; |z| \geq 1$$

F_a and f share same singularities $z_{k,p}$ inside D $2K$

Poles of approximants (t. 5)

f: finitely many poles *and* branchpoints $z_{k,p}$ in D — K of both...
smooth behaviour near T

Convergence results, deep potential theory: — $n \to \infty$, simple poles

the poles of R_n converge to these singularities — (weak sense, capacity)

Localisation results: — strong, also triple poles

when $f(z)$ close to $\varphi(z)/(z - z_{k,p})^3$ in $L^2(T)$, — section Π_p next to C_k

(φ analytic in D) — first poles of R_n accumulate to $z_{k,p}$

In 2D sections

Best quadratic rational approximation on T_p:

for $n \geq 0$, find polynomials π_n, q_n with degree $p_n \leq$ degree q_n and q_n with zeroes in D_p that minimize

$$\left\| f_p - \frac{\pi_n}{q_n} \right\|_{L^2(T_p)}$$

among such functions $\frac{\pi_n}{q_n} \in H^2_-(D_p)$

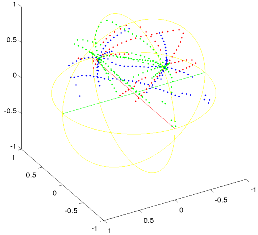
- increase degree n until error small enough on $T_p \rightsquigarrow K$...
- zeroes of q_n = poles of π_n/q_n, approximate singularities z_{kp} of f_p in D_p

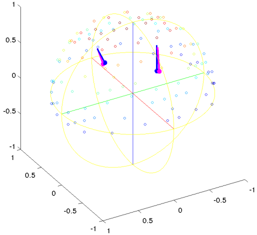
Even better here, get triple poles: minimize

$$\left\| f_p - \frac{\pi_n}{q_n^3} \right\|_{L^2(T_p)}$$

Pointwise sources recovery

Best L^2 rational approximation on plane sections
(circles, 12 directions) [software FindSources3D (matlab), APICS-ATHENA]

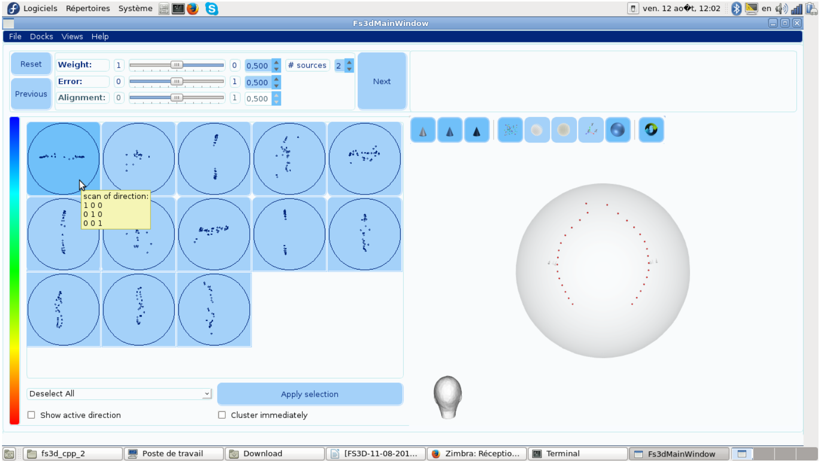


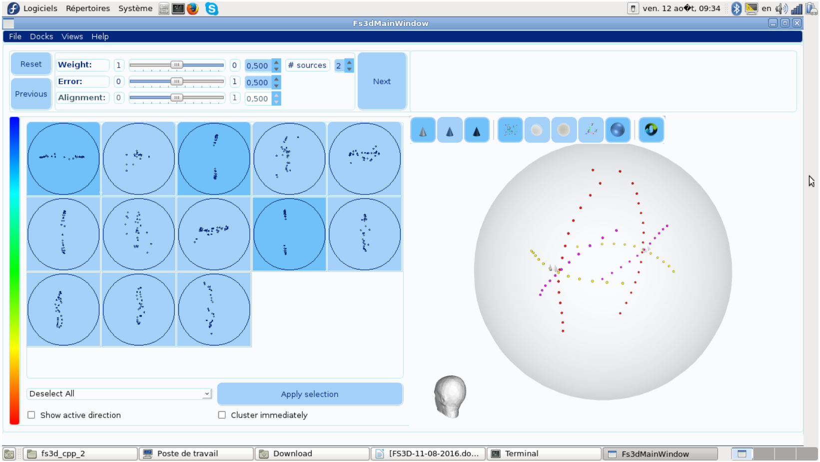


$K = 2$ sources, recovered with their moments

Other sources C_1, C_2

FS3D-bolis: modular ergonomical release





Logiciels Répertoires Système
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File Docks Views Help
Reset
Previous
Weight:
Error:
Alignment:
0,500
sources
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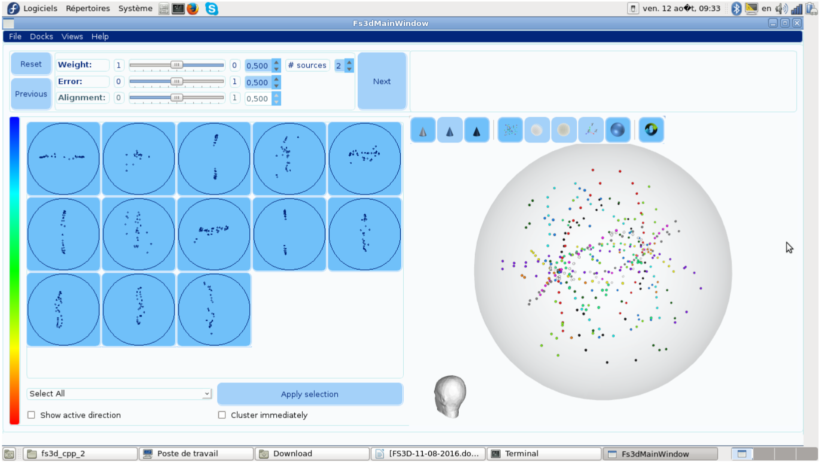
Terminal
Fs3dMainWindow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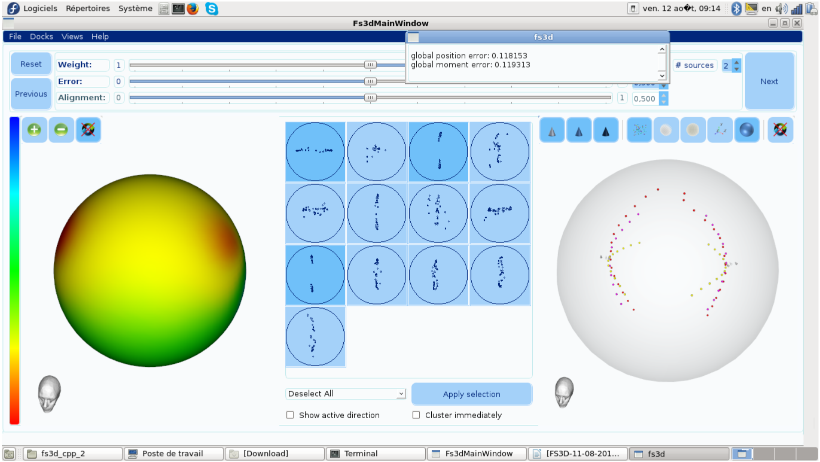

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Previous
Weight:
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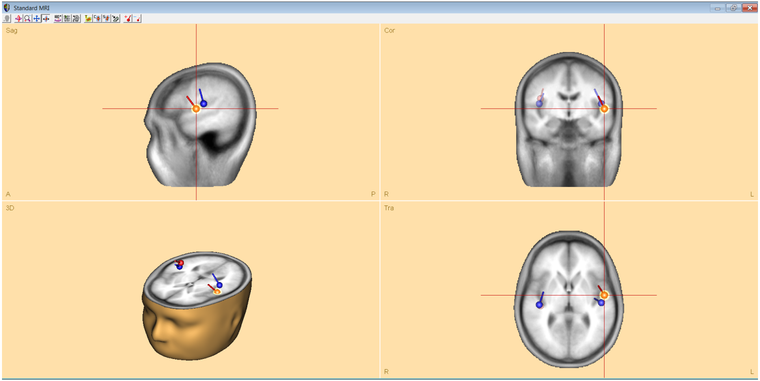
Error:
0
Alignment:
0
1
0,500
sources
2
Next
Deselect All
Apply selection
Show active direction
Cluster immediately
fs3d_cpp_2
Poste de travail
[Download]
Terminal
Fs3dMainWindow
[FS3D-11-08-201...
fs3d

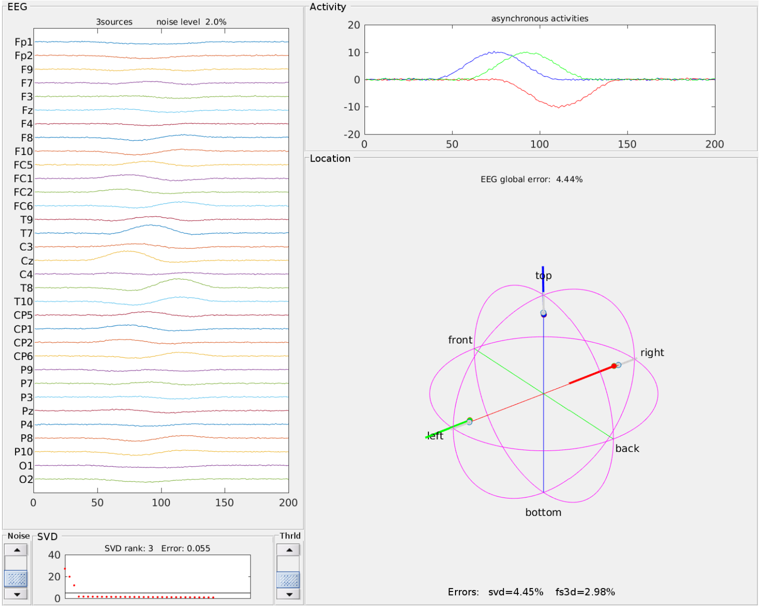
Actual heads

Actual and estimated sources and moments, from simulated data by BESA (with the courtesy of BESA GmbH)



FindSources3D: allows to find K and to localize time correlated sources

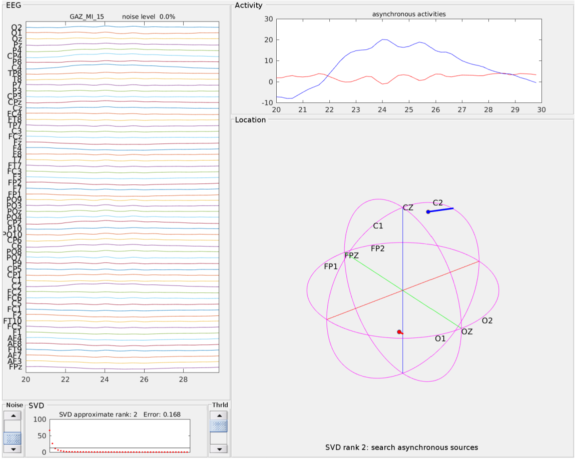
FindSources3D-μ



FindSources3D-μ

Evoked potentials

left wrist, hospital la Timone, Marseille



SVD, suitable combination of principal components $(|p_k| = |p_k(t)|)$

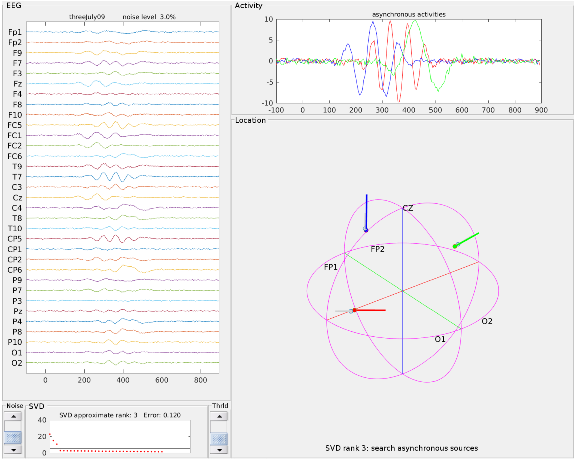
Main activity: right hemishpere

electrodes: O occiput, F frontal, ...

FindSources3D-μ

Other dataset

BESA GmbH, Germany



Perspectives

- FS3D-μ:
  - use MEG data (magnetoencephalography, la Timone hosp., Athena INRIA team)

    measures of normal component of magnetic field: $B_n(x) = \frac{x}{|x|} \cdot \sum_{k=1}^{K} \frac{p_k \times C_k}{|x - C_k|^3}$
  - Time window selection, more realistic geometries
- Conductivity estimation issues (EIT, Electrical Impededance Tomography)

  recover unknown conductivity values uniqueness issues, σ_1 skull [PhD CP]
- silent sources?
- other geometries? (ellipsoids, realistic)
- other elliptic operators (and PDEs)? Schrödinger, Helmholtz, time t
- approximation by $3D$ singular functions? $\nabla\ (1/|X - X_p|)$, $X_p \in \Omega$

Short bibliography

Clerc, Leblond, Marmorat, Papadopoulo, Source localization using rational approximation on plane sections (2012)

Baratchart, Leblond, Marmorat, Inverse sources problem in a 3D ball from best meromorphic approximation on 2D slices (2006)

Leblond, Paduret, Rigat, Zghal, Source localization in ellipsoids by best meromorphic approximation in planar sections (2008)

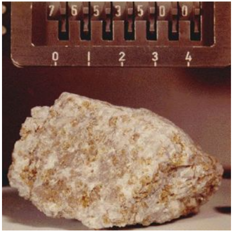
FindSources3D:
`http://www-sop.inria.fr/apics/FindSources3D/`

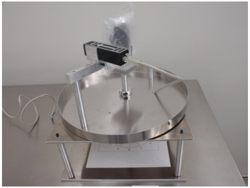
Applications to paleomagnetism

Inverse source estimation problems

with L. Baratchart, S. Chevillard, J.-P. Marmorat, K. Mavreas

Cerege-CNRS (Aix-en-Provence, ANR MagLune), Moon rocks (NASA), lunometer






Measures of magnetic field (magnetometer)

$\rightsquigarrow$ magnetization (in rock)?

$\rightsquigarrow$ past and future of the Earth magnetic field

Maxwell's equations (magnetostatics)

Quasi-static assumptions — **H** magnetic field

Ampère's law, no external current density ($\mathcal{J} = 0$):

$\nabla \times \mathbf{H} = 0 \Rightarrow \mathbf{H} = -\nabla u$ — u magnetic potential (scalar)

magnetic flux density **B**: $\nabla \cdot \mathbf{B} = 0$

with constitutive relation: $\mathbf{B} = \mu_0 \left(\mathbf{H} + \mathbf{M} \right)$ for magnetization **M**

μ_0 magnetic permeability

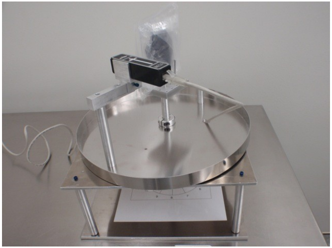
$$\Rightarrow \Delta u = \nabla \cdot \mathbf{M} = \operatorname{div} \mathbf{M}$$

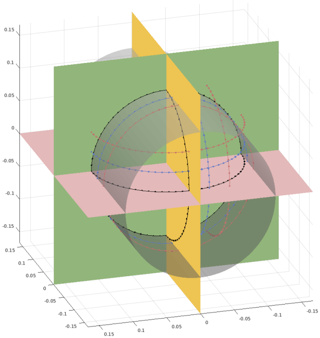
$\rightsquigarrow$ Laplace-Poisson PDE, yet with source term in div form

Inverse magnetization problems in Moon rocks

Mesures on magnetic field on surrounding circles in 3 $\perp$ directions

Lunometer at NASA, Cerege, CNRS [ANR]





$\rightsquigarrow$ estimate magnetic dipole in rock [PhD KM]

$$\Delta\, u = p \cdot \nabla \delta_C = \nabla \cdot \mathbf{M}\,, \;\; \mathbf{M} = p \delta_C$$

from values of field $\boldsymbol{B} = \nabla u\,, \;\; u(x) = \dfrac{\langle p\,,\, x - C \rangle}{|x - C|^3}$ on circles

Inverse source problems in magnetization

Planetary sciences, paleomagnetism, magnetic field of rocks samples $\rightsquigarrow$ magnetization?

- Moon rocks Cerege-CNRS
- meteorites, SQUID measurements, almost planar (thin) support of source term (distribution) **M** in 3D

  EAPS-MIT, Cambridge, MA, USA; associated Inria team Impinge,

  `http://www-sop.inria.fr/apics/IMPINGE/`

Baratchart, Hardin, Lima, Saff, Weiss, Characterizing kernels of operators related to thin-plate magnetizations via generalizations of Hodge decompositions (2013)

Baratchart, Chevillard, Leblond, Silent and equivalent magnetic distributions on thin plates, to appear

Futures

- Moon rocks: several dipoles [PhD KM]
- Inverse magnetization problems in small samples

  (Impinge associated team with MIT)

  - Net moment estimation (mean value of **M**)
  - Full magnetization recovery
  - 3D samples