Approximating Visibility Problems within a Constant

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Abstract

Visibility problems deal with placing a minimum number of transmitting stations in a region thus covering a maximum number of communication needs. Here we investigate some variants of such problems e.g. i) given a polygon P with weights on the vertices find at most k convex subpolygons C_i of P (possibly overlapping) with $V(C_i) \subseteq V(P)$ so that the weight of the vertices is a maximum, and ii) given a polygon (possibly with holes) and k available vertex (or edge) guards maximize a) the *length of boundary* guarded, b) the *total cost* of valuable parts of the boundary *watched (or covered)*. We give proofs of NP-hardness and also polynomial time algorithms that approximate the optimum within a constant ratio for the above problems. Furthermore we prove (for most of these problems) that they do not admit fully polynomial time approximation schemes, unless $P = NP$.

While investigating the above problems we introduce a) **weights** or values on pieces of the polygon's boundary, b) the useful and promising concept of **watching** a set of points or line segments as opposed to completely **overseeing** or covering it, and c) a way to discretize the boundary of the polygon by subdividing it into $O(n^2)$ pieces of the FVS = **finest visibility segmentation**, which is the finest relevant segmentation w.r.t. any geometrical consideration.

Keywords: Wireless Communication, Direct Point to Point Communication, Point to Station Communication, Approximation Algorithms, Visibility Problems, Computational Geometry, Visibility Graphs.

1 Introduction

The development of wireless communication technology (mobile phones, etc) created a number of research problems: minimization of the number of transmitting antennas, minimization of the number of used frequencies, etc. Two points can communicate if they are covered by (i.e. can communicate with) an antenna. Notice that communication networks, use such high frequency ranges that side effects of reflection and refraction become important unless the two points are mutually visible (on a straight line segment). Thus a straight line of sight approach models reality with sufficient precision. The well known problem is how to place stations so that all points are covered (visible) and the number of stations is minimum. A variation is: a number of stations is given, and we are asked to cover as many points as possible. Whether communication between two points is possible or blocked it depends on the area topology. So the model must keep the properties of the topology. Graphs, terrains, polygons with (without) holes, etc, have been used as models. In general topologies most of these problems are NP-hard and many of them are APX-hard or even worse. In a more restricted model we can usually do better. Such an example is a visibility graph: its vertex set is the vertex set of a polygon and two vertices share an edge iff they are mutually visible in the polygon. Recognition of a visibility graph is in PSPACE [4]. The visibility graph is an interesting representation model because a number of visibility problems for polygons correspond to graph problems (e.g. a maximum convex subpolygon C of a polygon P with vertex set $V(C) \subseteq V(P)$, corresponds to a maximum clique in the visibility graph of P). There are problems that in visibility graphs are easier than in general graphs (e.g. maximum clique). Another example of a restricted model is a polygon with holes. The points that must be covered lie in general on the boundary of the polygon and of its holes. On the other hand the covering stations may be vertices or whole edges of the polygon. Thus, covering stations may be placed in the interior of the polygon, i.e. on vertices or edges of holes. A polygon with holes is a quite general topology. In fact, for every graph we can easily construct a polygon with holes in which two vertices are mutually visible iff they share an edge in the graph. We study a number of visibility problems for polygons with (without) holes. (Visibility problems are sometimes known as art gallery problems [2, 5, 6].) Some related problems that have been studied: MINIMUM VERTEX/EDGE/POINT GUARD for polygons with (without) holes (known to be APX-hard and $O(\log n)$ approximable [1, 7, 8]), MINIMUM FIXED HEIGHT VERTEX/POINT GUARD ON TERRAIN (best approximation possible $\theta(\log n)$ [7, 8, 10]), MAXIMUM WEIGHTED CLIQUE ON VISIBILITY GRAPH (known to be in P [13, 14, 15]), MINIMUM CLIQUE PARTITION ON VISIBILITY GRAPH for polygons without holes (known to be APX-hard and $O(\log n)$ approximable [7]). More specifically, we study: a) the problem of finding k cliques in the visibility graph of a polygon without holes so that the total weight of the clique vertices is maximum (MAXIMUM WEIGHT IN k CLIQUES), b) the following families of problems: given a number of available vertex (edge) guards: i) cover a maximum portion of the boundary of a polygon with (without) holes (MAXIMUM LENGTH VERTEX/EDGE GUARD), ii) watch (cover) a maximum total value of valuable portions of the boundary of a polygon with (without) holes (MAXIMUM VALUE VERTEX/EDGE GUARD). We prove that all of the above are NP-hard. We give for all of them polynomial time approximation algorithms achieving constant ratios, based on a well known greedy algorithm which approximates the MAXIMUM COVERAGE problem. Finally we prove that the MAXIMUM WEIGHT IN k CLIQUES and MAXIMUM VALUE VERTEX/EDGE GUARD do not admit a FPTAS unless P=NP (i.e. if P$\neq$NP).

Notation and Preliminaries

Definition 1 *Let P be a polygon with (without) holes, $V = (v_0, v_1, \ldots, v_{n-1})$ its vertices, $E = (e_0, e_1, \ldots, e_{n-1})$ its edges and $\partial(P)$ its boundary. Let $a, b \in P$, be points and $L, M \subseteq P$ sets of points. We define the following predicates:*

1. *sees(a,b): the straight line segment connecting a and b lies (everywhere) inside P. Note that: sees$(a,b) \leftrightarrow$ sees(b,a).*
2. *oversees(M,L): $\forall a \in L\ \exists b \in M : sees(a,b)$. We say that L is* visible *from M or that M* covers *L. Note that oversees(M,L) is not symmetric.*
3. *watches(M,L): $\exists a \in L\ \exists b \in M : sees(a,b)$. Note that watches$(M,L)$ is symmetric.*

Definition 2 *Let P be a polygon with (without) holes. The problem (boundary restricted)* MINIMUM VERTEX/EDGE GUARD *is the problem of finding a minimum subset S of vertices (edges) of P such that $\partial(P)$ is visible from S [oversees$(S, \partial(P))$]. The vertices (edges) in S are called Vertex (respectively edge) Guards.*

Our approximation algorithms are based on the following well known NP-hard problem:

Definition 3 *Given is a universe set U with weighted elements, an integer $k > 0$ and a collection C of subsets of U. The* MAXIMUM COVERAGE *problem asks for k sets $S_i \in C$ s.t. $\bigcup S_i$ has maximum total weight.*

Algorithm 1 MaxCoverage (* greedy *)

```
SOL ← ∅
for i = 1 to k do
    select S_i ∈ C that maximizes Weight(SOL ∪ S_i)
    SOL ← SOL ∪ S_i
end for
return Weight(SOL)
```

Theorem 1 *[16, 17, 18, 12] Algorithm 1 runs in polynomial time and approximates the* MAXIMUM COVERAGE *problem achieving a $0.632 \simeq 1 - \frac{1}{e}$ [actually $1-(1-\frac{1}{k})^k$] ratio.*

2 The MAXIMUM WEIGHT IN k CLIQUES problem

Given is a polygon P without holes, with weights on its vertices and an integer $k > 0$. We are asked for k convex subpolygons C_i of P (possibly overlapping) with $V(C_i) \subseteq V(P)$ so that the weight of the vertices is a maximum. Although this problem is of great theoretical interest, here is an application in wireless communication networks: Given is a number of villages with their populations, modelled as weighted vertices of a simple polygon without holes. We are asked to design at most k wireless communication networks, so that a maximum number of people can communicate and all villages in the same network can communicate directly and not through a station. In the abstract version, instead of a polygon, a graph is given along with a polynomial time checkable proof that this graph is the visibility graph of some polygon P (e.g. the polygon P itself).

Definition 4 *Given is a polygon P without holes, with weights on its vertices and an integer $k > 0$. The goal of the* MAXIMUM WEIGHT IN k CLIQUES *problem is to find up to k cliques in the visibility graph of P, so that the weight of covered vertices is maximum.*

There is a number of related problems that have been studied:

Definition 5 *Given is a polygon P without holes. The goal of the* MINIMUM CONVEX DECOMPOSITION *problem is to partition the polygon to a minimum number of non-overlapping convex polygons.*

It is known [19, 20] that the MINIMUM CONVEX DECOMPOSITION problem can be solved in polynomial time.

Definition 6 *Given is a polygon P without holes. The goal of the* MINIMUM CONVEX COVER *problem is to cover the polygon using a minimum number of (possibly overlapping) convex polygons that lie inside P.*

It is known [7] that the MINIMUM CONVEX COVER problem is APX-hard. A logarithmic approximation algorithm is also known [7] for this problem.

We used the following problem to establish NP-hardness for our problem:

Definition 7 *Given is a polygon P without holes. The goal of the* MINIMUM CLIQUE PARTITION ON VISIBILITY GRAPH *problem is to partition the visibility graph of P to a minimum number of cliques.*

The MINIMUM CLIQUE PARTITION ON VISIBILITY GRAPH problem is APX-hard [7]. A logarithmic approximation algorithm exists for the problem.

One can easily see that the decision version of the MINIMUM CLIQUE PARTITION ON VISIBILITY GRAPH problem Karp-reduces to the decision version of MAXIMUM WEIGHT IN k CLIQUES. Thus:

Fact 1 MAXIMUM WEIGHT IN k CLIQUES *is NP-hard.*

In our approximation algorithm we will use the following problem:

Definition 8 *Given is a polygon P without holes, with weights on its vertices. The goal of the* MAXIMUM WEIGHTED CLIQUE ON VISIBILITY GRAPH *problem is to find a maximum weight clique on the visibility graph of polygon P.*

The MAXIMUM WEIGHTED CLIQUE ON VISIBILITY GRAPH problem is in P [13, 14, 15]. An algorithm that solves MAXIMUM WEIGHTED CLIQUE ON VISIBILITY GRAPH in $O(n^3)$ has been described in [7].

Algorithm 2 MaximumWeightinkCliques (* greedy *)

$SOL \leftarrow \emptyset$; $W_S \leftarrow W$
for $i = 1$ to k **do**
  $S_i \leftarrow$ MaxWeightedClique(V_G)
  $SOL \leftarrow SOL \cup V(S_i)$
  $W_s(V(S_i)) \leftarrow 0$
end for
return $W(SOL)$

Proposition 1 *Algorithm 2 runs in polynomial time and achieves a* $0,632$ *constant approximation ratio of the optimum of the* MAXIMUM WEIGHT IN k CLIQUES *problem.*

Proof. We find at each step the MAXIMUM WEIGHTED CLIQUE ON VISIBILITY GRAPH of the polygon, taking as weights the updated ones in W_s. Thus, at each iteration we take into the solution a set which causes a maximum increase to the overall weight, similar to Algorithm 1. □

Theorem 2 MAXIMUM WEIGHT IN k CLIQUES *does not admit a FPTAS, unless $P = NP$.*

Proof. Suppose there exists a FPTAS for the problem. We will show that the decision version of MINIMUM CLIQUE PARTITION ON VISIBILITY GRAPH can be decided in polynomial time and thus P=NP, because the decision version of MINIMUM CLIQUE PARTITION ON VISIBILITY GRAPH is known to be NP-complete. So, we suppose there exists a polynomial time (w.r.t. input size and w.r.t. $\frac{1}{\varepsilon}$) approximation algorithm that achieves an approximation ratio of $1-\varepsilon,\ \forall \varepsilon > 0$. Consider the decision version of MINIMUM CLIQUE PARTITION ON VISIBILITY GRAPH of a polygon with n vertices. The question to be decided is whether the visibility graph can be partitioned into $\leq k$ cliques. For the (Karp) transformation to our problem we keep the same visibility graph, and we assign weight 1 to all vertices. We use the FPTAS with $\varepsilon = \frac{1}{n}$ to find the maximum weight of vertices that belong to k cliques. Let SOL be the solution of the FPTAS and OPT an optimal solution. Now MINIMUM CLIQUE PARTITION ON VISIBILITY GRAPH can be decided, namely:

1. if $SOL = n$ then $OPT = n$, and thus the answer is "yes".
2. if $SOL \leq n-1$ then by the existence of a FPTAS:

$$(1-\frac{1}{n})OPT = (1-\varepsilon)OPT < SOL$$

and thus

$$(1-\frac{1}{n})OPT < n-1, \text{ i.e. } OPT < n,$$

and the answer is "no" (i.e. the graph cannot be partitioned into k cliques).

Therefore we have an answer in any case for the MINIMUM CLIQUE PARTITION ON VISIBILITY GRAPH problem in $\mathtt{Poly}(n+\frac{1}{\varepsilon}) = \mathtt{Poly}(n)$ time. □

3 The MAXIMUM LENGTH VERTEX/EDGE GUARD problem for polygons with (without) holes

Suppose a polygon P with (without) holes is given. We are asked to cover a maximum portion of the polygon's boundary (possibly including boundaries of the holes), using no more than k stations. We are allowed to use either only vertex stations or only edge stations (occupying whole edges).

Definition 9 *Given is a polygon P with (without) holes and an integer $k > 0$. Let $L(b)$ be the euclidean length of the line segment b. The goal of the* MAXIMUM LENGTH VERTEX/EDGE GUARD *problem is to place k vertex (edge) guards (stations) so that the euclidean length of that part of P's boundary that is* overseen *(covered) by the guards is maximum.*

It is known that MINIMUM VERTEX/EDGE GUARD for polygons with (without) holes is NP-hard [1] and that it admits polynomial time approximation algorithms which achieve $O(\log n)$ approximation ratios [10].

Furthermore it is proved in [8, 7] that MINIMUM VERTEX/EDGE GUARD for polygons without holes is APX-hard. For polygons with holes, it is proved in [9, 7] that no polynomial time approximation algorithm can guarantee an approximation ratio of $\frac{1-\varepsilon}{12}\ln n$ for any $\varepsilon > 0$, unless $NP \subseteq TIME(n^{O(\log\log n)})$. Observing that the decision version of MINIMUM VERTEX/EDGE GUARD for polygons with (without) holes Karp-reduces to the corresponding decision version of MAXIMUM LENGTH VERTEX/EDGE GUARD, for polygons with (without) holes, we can easily generalize:

Fact 2 MAXIMUM LENGTH VERTEX/EDGE GUARD *for polygons with (without) holes is NP-hard.*

Algorithm 3 approximates the MAXIMUM LENGTH VERTEX/EDGE GUARD problem for polygons with or without holes using $E'(v)$ (or $E'(e)$) which is the set of line segments on the boundary visible from v (or e). To construct $E'(v)$ (or $E'(e)$), we use the visibility graph $V_G(P)$. By extending edges of $V_G(P)$ inside P up to the boundary of P we obtain a set of points FVS of the boundary of P (that includes of course all vertices) (see figure 1). There are $O(n^2)$ points in FVS (= finest visibility segmentation) and these points are endpoints of line segments with the following property: for any point $y \in FVS$, a segment (a,b) defined by consecutive FVS points a, b is visible by y iff it is watched by y. Furthermore (a,b) is visible by an edge e iff it is visible by any point in $FVS \cap e$. Thus we can find the set of line segments $E'(v)$ ($E'(e)$)which are visible by a vertex v (edge e) within time $O(n^2)$ ($O(n^4)$).

Algorithm 3 MaxLegthVertex/EdgeGuards (* greedy *)

$SOL \leftarrow \emptyset$
for $i = 1$ to k **do**
  select $x \in V$ $(x \in E)$ that maximizes $L(SOL \cup E'(x))$
  $SOL \leftarrow SOL \cup E'(x)$
end for
return $L(SOL)$

Proposition 2 *Algorithm 3 runs in polynomial time and achieves a constant* $0,632$ *approximation ratio of the optimum of the* MAXIMUM LENGTH VERTEX/EDGE GUARD *problem.*

Proof. As in Algorithm 2, at each iteration, a maximum increase to the overall solution is achieved. Thus our polynomial time algorithm achieves a 0.632 approximation ratio. □

In [21] we prove that MAXIMUM LENGTH VERTEX/EDGE GUARD for polygons with or without holes is APX-hard. Furthermore, in [22], we extend the definition of the MAXIMUM LENGTH VERTEX/EDGE GUARD problem by allowing guard (station) placement in the interior of the polygon with the goal to cover a maximum part of the interior area of the polygon. We give constant approximation algorithms and prove APX-hardness.

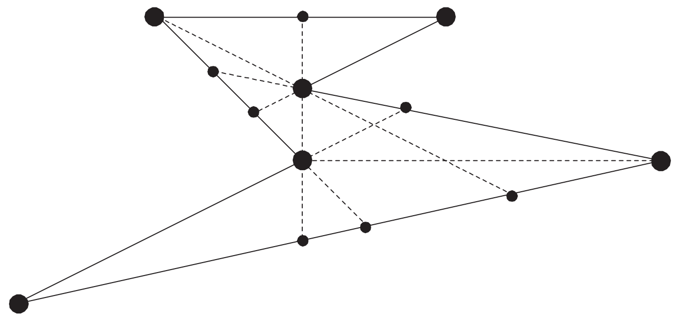

Figure 1: Subdividing the boundary into line segments with endpoints in FVS

4 The MAXIMUM VALUE VERTEX/EDGE GUARD problem

Suppose a polygon P with (without) holes is given with weighted disjoint line segments on its boundary (including boundaries of the possible holes). Our line segments are open intervals (a,b). A possible interpretation of line segments is disjoint districts. Weights may be interpreted as population numbers. Another interpretation for line segments is paintings in an art gallery. Weights may be interpreted as cost values. We are asked to cover a maximum weight using no more than k stations. As before, we have two versions w.r.t the type of stations: a) vertex stations or b) edge stations.

Definition 10 *Given is a polygon P with (without) holes and an integer* $k > 0$*. Assume the boundary of P is subdivided into disjoint line segments with non negative weights (see figure 2). The goal of the* MAXIMUM VALUE VERTEX/EDGE GUARD *problem is to place k vertex (edge) guards (e.g. stations) so that the total weight of the set of line segments watched (overseen) is maximum.*

Proposition 3 MAXIMUM VALUE VERTEX/EDGE GUARD*, for polygons with (without) holes is NP-hard.*

Proof. The decision version of MINIMUM VERTEX/EDGE GUARD for a polygon P with (without) holes Karp-reduces to the corresponding decision version of

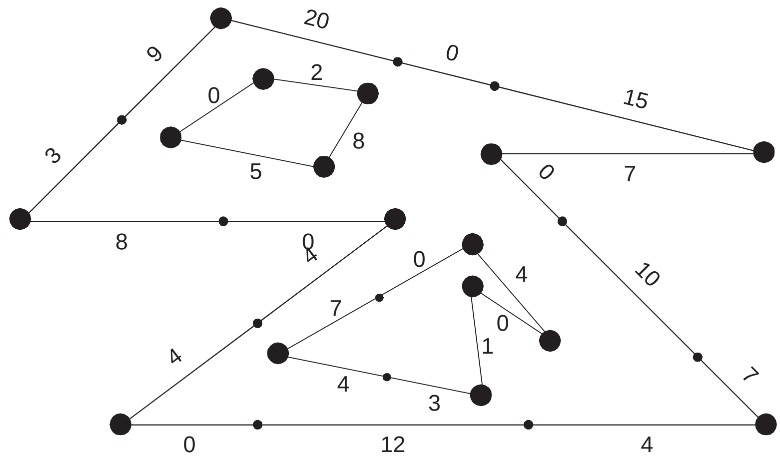

Figure 2: A weighted polygon

MAXIMUM VALUE VERTEX/EDGE GUARD for the same polygon P with (without) holes. We construct the FVS, i.e. the finest line segment subdivision of the edges of P by using $E'(v_i)$ $(E'(e_i))$ of the previous section for all v_i (e_i). Each edge is subdivided into non overlapping segments (see figure 1). Every segment of E' is thus *watched* iff it is *visible* by a vertex (edge). We assign weight 1 to every segment. Let T be the total weight (note that every piece of the boundary is visible by some vertex (edge)). Thus the original polygon boundary is k-guardable iff the new weighted polygon is k-watched (-visible) with total weight T. □

Algorithm 4 approximates the MAXIMUM VALUE VERTEX/EDGE GUARD problem for polygons with or without holes: For the case of vertex (edge) watching guards for polygons with (without) holes , it is easy to calculate at each iteration the set of segments $S(v)$ $(S(e))$ which are watched by v (e). For the case of vertex (edge) overseeing guards for polygons with (without) holes , we calculate $E'(v)$ $(E'(e))$ for every vertex (edge) and then we calculate the total weight of line segments to be included in $SOL \cup E'(v)(E'(e))$.

Proposition 4 *Algorithm 4 runs in polynomial time and achieves a* 0.632 *constant approximation ratio of the optimum of the* MAXIMUM VALUE VERTEX/EDGE GUARD *problem.*

Proof. As in Algorithm 3, at each iteration, a maximum increase to the overall solution is achieved. Thus our polynomial time algorithm achieves a 0.632 approximation ratio. □

Algorithm 4 MaxValueVertex/EdgeGuards (* greedy *)

$SOL \leftarrow \emptyset$
for $i = 1$ to k **do**
  select $x \in V$ $(x \in E)$ that maximizes $\texttt{Weight}(SOL \cup S(x)(orE'(x))$
  $SOL \leftarrow SOL \cup S(x)(orE'(x))$
end for
return $\texttt{Weight}(SOL)$

Theorem 3 MAXIMUM VALUE VERTEX/EDGE GUARD *for polygons with (without) holes does not admit a FPTAS, unless* $P = NP$.

Proof. Suppose there exists a FPTAS for the problem. We will show that the decision version of MINIMUM VERTEX/EDGE GUARD can be decided in polynomial time and thus P=NP, because the decision version of MINIMUM VERTEX/EDGE GUARD is known to be NP-complete. So, we suppose there exists a polynomial time (w.r.t. input size and w.r.t. $\frac{1}{\varepsilon}$) approximation algorithm that achieves an approximation ratio of $1 - \varepsilon$, $\forall \varepsilon > 0$. Consider the decision version of MINIMUM VERTEX/EDGE GUARD of a polygon P with n vertices. The question to be decided is whether P can be guarded using $\leq k$ guards. For the (Karp) transformation to our problem we keep the same polygon, and we assign weight 1 to all segments. Let $M = |FVS|$. We use the FPTAS with $\varepsilon = \frac{1}{M}$ to find a segment set with maximum weight that can be watched or overseen by k guards. Let SOL be the solution of the FPTAS and OPT an optimal solution. Now MINIMUM VERTEX/EDGE GUARD can be decided, namely:

1. if $SOL = M$ then $OPT = M$, and thus the answer is "yes".
2. if $SOL \leq M - 1$ then by the existence of a FPTAS:
$$(1 - \frac{1}{M})OPT = (1 - \varepsilon)OPT < SOL$$
and thus
$$(1 - \frac{1}{M})OPT < M - 1, \texttt{ i.e. } OPT < M,$$
and the answer is "no" (i.e. P cannot be guarded using $\leq k$ guards).

Therefore we have an answer in any case for the MINIMUM VERTEX/EDGE GUARD problem in $\texttt{Poly}(n + \frac{1}{\varepsilon}) = \texttt{Poly}(n)$ time. □

We prove in [21] that watching or overseeing cases of the MAXIMUM VALUE VERTEX/EDGE GUARD problem for polygons with or without holes are APX-hard. Furthermore, in [22], we extend the definition of MAXIMUM VALUE VERTEX/EDGE GUARD by allowing guards to be placed in the interior of the polygon.

We also extend the intended guarded area to the interior of the polygon (i.e. covering maximum valuable parts of the interior). We give constant approximation algorithms and prove APX-hardness.

5 Conclusion

We investigated the following problems: 1) MAXIMUM WEIGHT IN k CLIQUES for visibility graphs of polygons without holes, 2) MAXIMUM LENGTH VERTEX GUARD for polygons without holes, 3) MAXIMUM LENGTH VERTEX GUARD for polygons with holes, 4) MAXIMUM LENGTH EDGE GUARD for polygons without holes, 5) MAXIMUM LENGTH EDGE GUARD for polygons with holes, 6) watching MAXIMUM VALUE VERTEX GUARD for polygons without holes, 7) watching MAXIMUM VALUE VERTEX GUARD for polygons with holes, 8) watching MAXIMUM VALUE EDGE GUARD for polygons without holes, 9) watching MAXIMUM VALUE EDGE GUARD for polygons with holes, 10) overseeing MAXIMUM VALUE VERTEX GUARD for polygons without holes, 11) overseeing MAXIMUM VALUE VERTEX GUARD for polygons with holes, 12) overseeing MAXIMUM VALUE EDGE GUARD for polygons without holes, 13) overseeing MAXIMUM VALUE EDGE GUARD for polygons with holes. We proved NP-hardness and we found polynomial time approximation algorithms with constant ratio for all of them using the greedy technique that approximates the MAXIMUM COVERAGE problem. We also proved that most of them do not admit a FPTAS, unless $P = NP$. While investigating the above problems we introduced a) **weights** or values on pieces of the polygon's boundary, b) the useful and promising concept of **watching** a set of points or line segments as opposed to completely **overseeing** or covering it, and c) a way to discretize the boundary of the polygon by subdividing it into $O(n^2)$ pieces of the FVS = **finest visibility segmentation**, which is the finest relevant segmentation w.r.t. any geometrical consideration. Furthermore the FVS satisfies the following interesting property: a FVS line segment (a,b) is watched by a FVS point y iff (a,b) is visible by y.

We extend our results in [22] for all these problems, by introducing costs on candidate station places (vertices, edges or interior points) and a budget B. The goal is to maximize the value of the guarded places by positioning guards that cost totally at most B. We give constant approximation algorithms and prove APX-hardness.

References

[1] Lee, D., Lin, A., Computational complexity of art gallery problems, IEEE Trans. Inform. Theory 32, 276-282, 1986.

[2] O'Rourke, J., Art Gallery Theorems and Algorithms, Oxford University Press, New York, 1987.

[3] Shmoys, D., Tardos, E., An approximation algorithm for the generalized assignment problem, Mathematical Programming A, 62:461-74, 1993.

[4] Everett, H., Visibility graph recognition, *Report 231/90, Dept. Comput. Sci., Univ. Toronto*, Toronto, ON, 1990, Ph.D. Thesis.

[5] Shermer, T., Recent results in Art Galleries, Proc. of the IEEE, 1992.

[6] Urrutia, J., Art gallery and Illumination Problems, Handbook on Computational Geometry, 1998.

[7] Eidenbenz, S., (In-)Approximability of Visibility Problems on Polygons and Terrains, PhD Thesis, ETH Zurich, 2000.

[8] Eidenbenz, S., Inapproximability Results for Guarding Polygons without Holes, Lecture notes in Computer Science, Vol. 1533 (ISAAC'98), p. 427-436, 1998.

[9] Eidenbenz, S., Stamm, C., Widmayer, P.,Inapproximability of some Art Gallery Problems, Proc. 10th Canadian Conf. Computational Geometry (CCCG'98), pp. 64-65, 1998.

[10] Ghosh, S., Approximation algorithms for Art Gallery Problems, Proc. of the Canadian Information Processing Society Congress, pp. 429-434, 1987.

[11] Hochbaum, D., Approximation Algorithms for NP-Hard Problems, PWS Publishing Company, 1996.

[12] Hochbaum, D., Approximating Covering and Packing Problems in: Approximation Algorithms for NP-Hard Problems (ed. Dorit Hochbaum), pp. 94-143, PWS Publishing Company, 1996.

[13] Avis, D., Rappaport, D., Computing the largest empty convex subset of a set of points, Proc. 1st Ann. ACM Symposium Computational Geometry, pp. 161-167, 1985.

[14] Dobkin, D., Edelsbrunner, H., Overmars, H., Searching for Empty Convex Polygons, Algorithmica 5, pp. 561-571, 1990.

[15] Edelsbrunner, H., Guibas, L., Topologically sweeping an arrangement, J. Comput. System Sci. 38, pp. 165-194, 1989.

[16] Nemhauser, G., Wolsey, L., Maximizing submodular set functions: formulations and analysis of algorithms. In Studies of Graphs and Discrete Programming North-Holland, Amsterdam, 279-301, 1972.

[17] Conforti, M., Cornuejols, G., Submodular functions, matroids and the greedy algorithm: tight worst case bounds and some generalizations of the Rado-Edmonds theorem, Discrete Applied Mathematics 7:257-275, 1984.

[18] Vohra, R., Hall, N., A probabilistic analysis of the maximal covering location problem, Discrete Applied Mathematics 43:175-183, 1993.

[19] Chazelle, B., Dobkin, D., Decomposing a polygon into its convex parts, Proc. 11th Annual ACM Symp. Theory Comput., pp 38-48, 1979.

[20] Chazelle, B., Dobkin, D., Optimal convex decompositions, Computational Geometry, North-Holland, Netherlands, 1985.

[21] Markou, E., Zachos, S., Fragoudakis, C., Maximizing the Guarded Boundary of an Art Gallery is APX-hard, (submitted).

[22] Fragoudakis, C., Markou, E., Zachos, S., Budgeted Coverage of a Maximum Part of a Polygonal Area, (submitted).

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